

Magnetism basics



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<http://neel.cnrs.fr>

Micro-NanoMagnetism team : <http://neel.cnrs.fr/mnm>

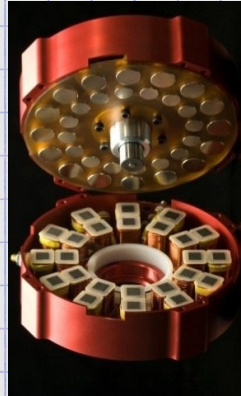




Modern applications of magnetism

Materials

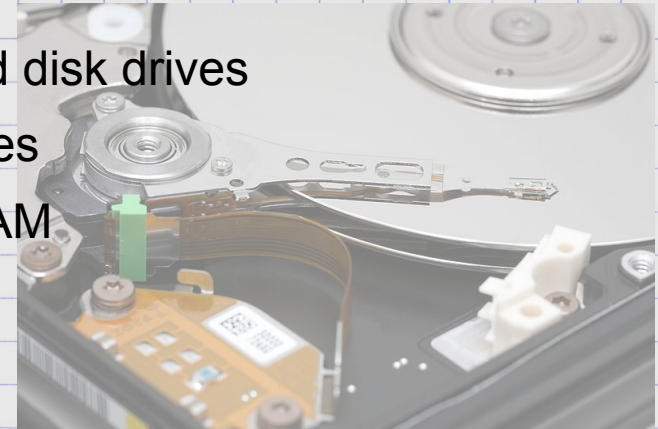
- ⇒ Magnets
(→ motors and generators)
- ⇒ Transformers
- ⇒ Magnetocaloric



Where does 'nano' contribute ?

Data storage

- ⇒ Hard disk drives
- ⇒ Tapes
- ⇒ MRAM



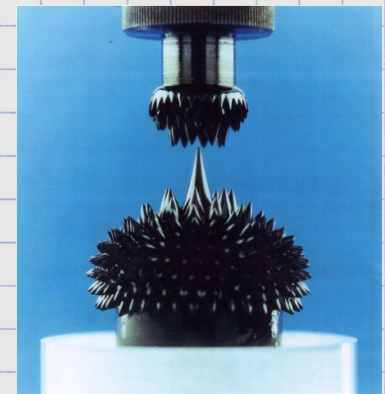
Sensors

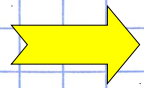
- ⇒ Compass
- ⇒ Field mapping
- ⇒ HDD read heads



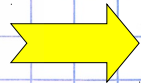
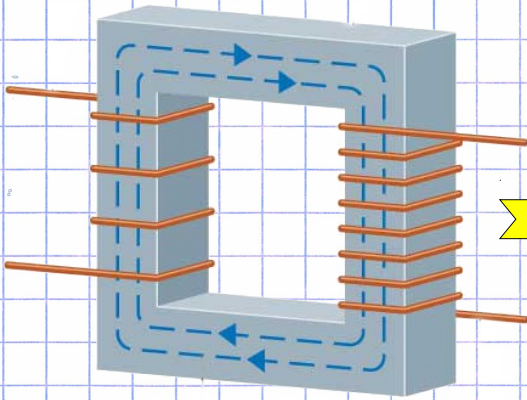
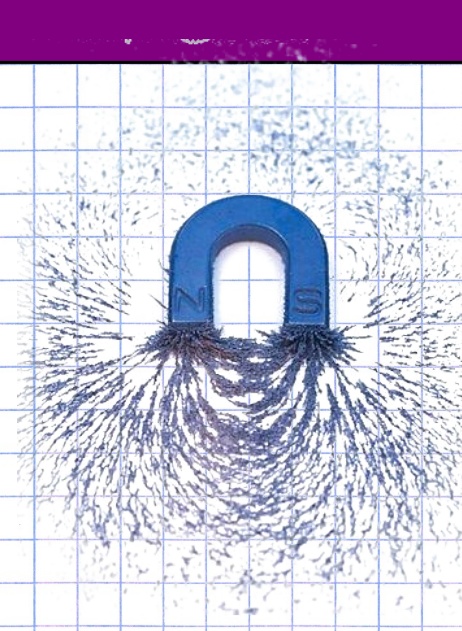
Nanoparticles

- ⇒ Ferrofluids
- ⇒ MRI contrast
- ⇒ Hyperthermia
- ⇒ Sorting & tagging

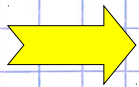
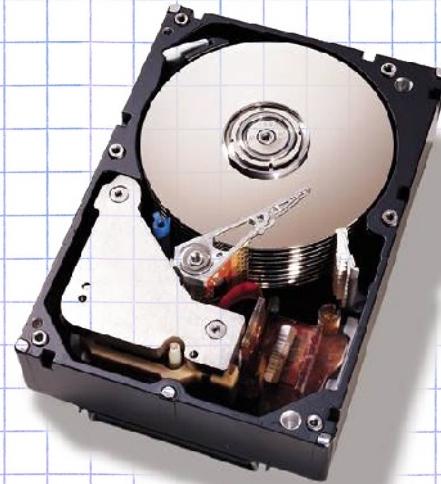




1. Fields, moments, materials



2. Magnetization processes

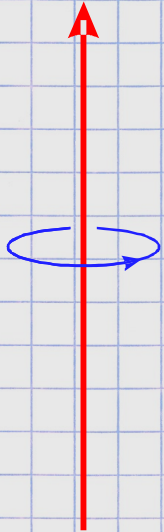


3. Thin film effects

Maxwell equation : $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$

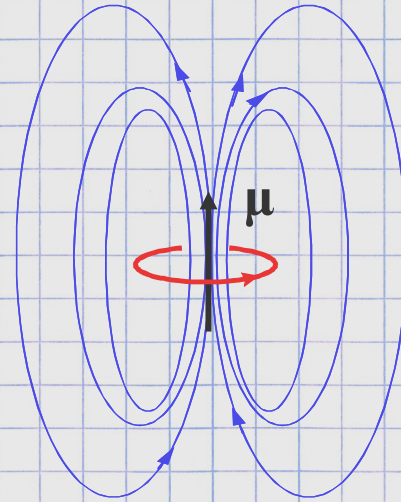
Electric currents produce magnetic fields

Oersted field



$$B_\theta = \frac{\mu_0 I}{2\pi r}$$

Magnetic dipole

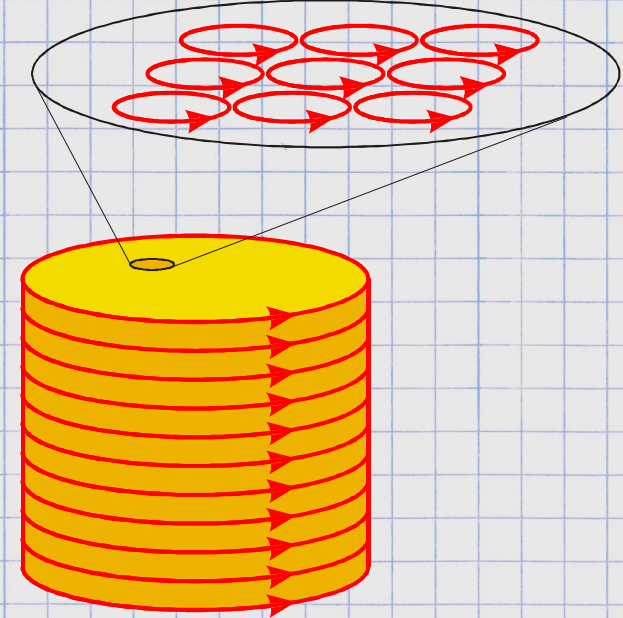


$$\mathbf{B} = \frac{\mu_0}{4\pi r^3}$$

$$\times \left[\frac{3}{r^2} (\boldsymbol{\mu} \cdot \mathbf{r}) \mathbf{r} - \boldsymbol{\mu} \right]$$

Magnetic dipole A.m^2

Magnetic material



Magnetization A/m

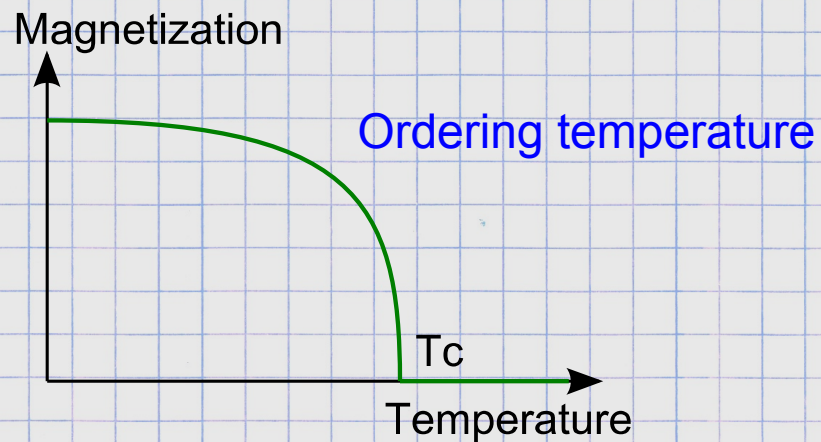
Magnetic moment A.m^2

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \quad \text{Induction } \mathbf{B}, \text{ Magnetic field } \mathbf{H}, \text{ Magnetization } \mathbf{M}$$

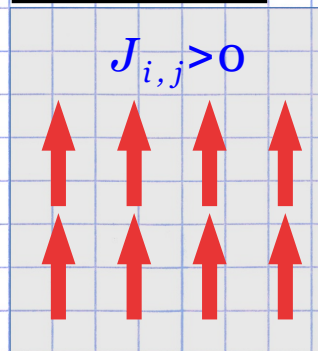
Magnetic ordering

Magnetic exchange
between microscopic moments

$$\mathcal{E} = -2 \sum_{i>j} J_{i,j} \mathbf{S}_i \cdot \mathbf{S}_j$$



Ferromagnet

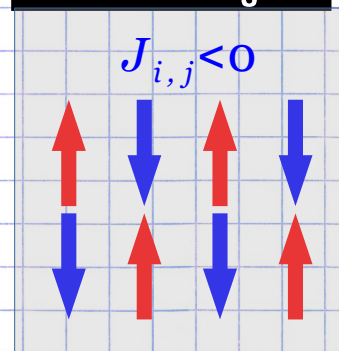


→ Fe

$T_c = 1043 \text{ K}$

$M_s = 1.73 \times 10^6 \text{ A/m}$

Antiferromagnet

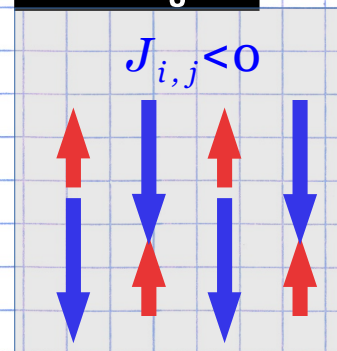


→ CoO

$T_N = 292 \text{ K}$

$J = 3/2$

Ferrimagnet

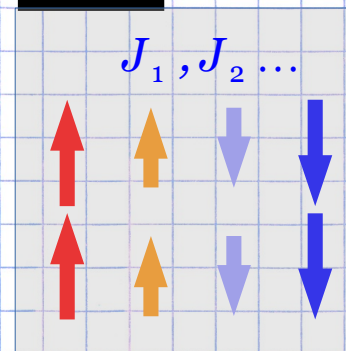


→ Fe₃O₄

$T_c = 858 \text{ K}$

$M_s = 480 \text{ kA/m}$

Helical



→ Dy

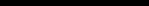
$T \in 85 - 179 \text{ K}$

$\mu = 10.4 \mu_B$

H																	He
Li	Be											B	C	N	O	F	Ne
Na	Mg											Al	Si	P	S	Cl	Ar
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba		Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra		Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Uub	Uut	Uuq	Uup	Uuh	Uus	Uuo
		La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu	
		Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr	

Antiferromagnetic
Diamagnetic
Ferromagnetic
Paramagnetic

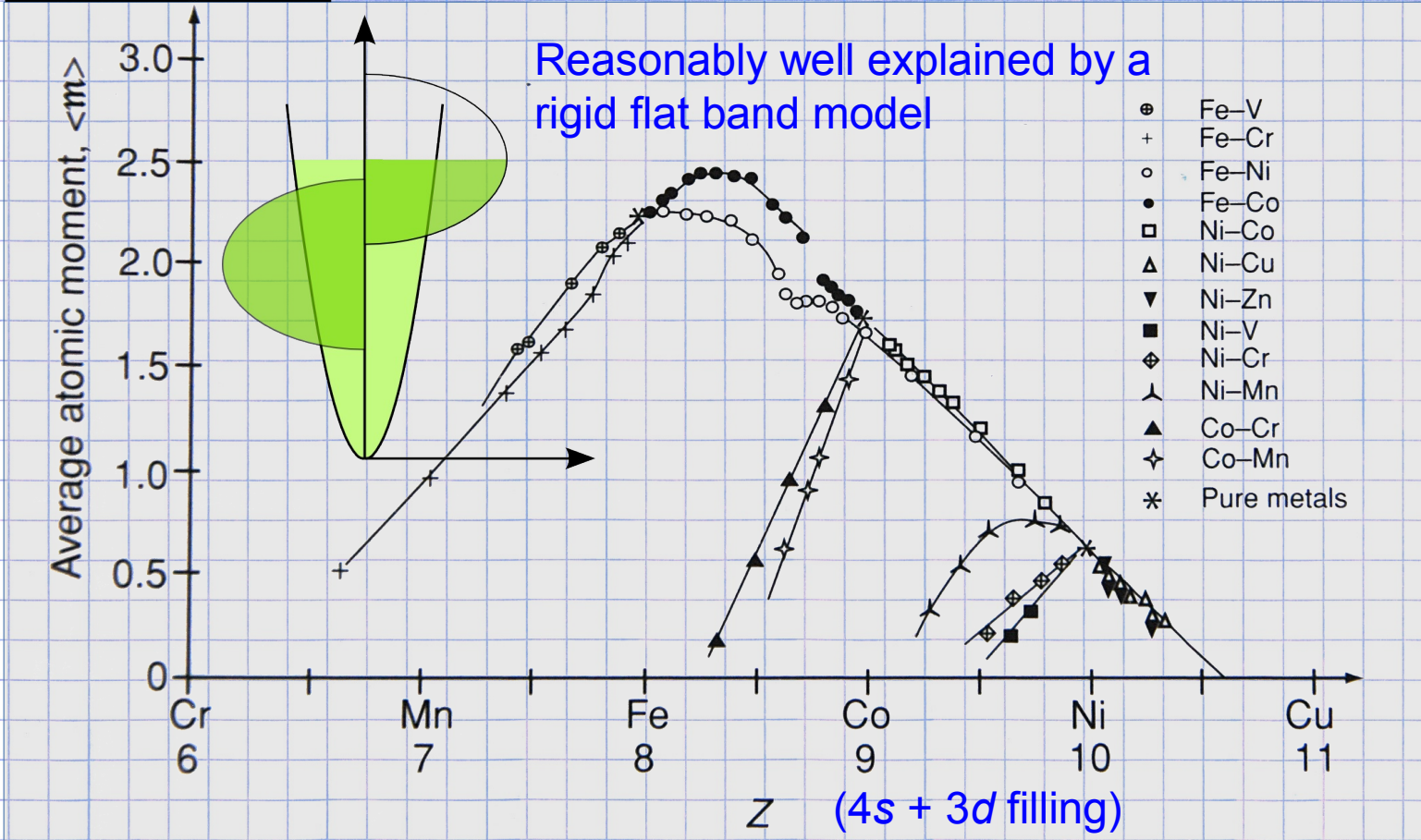
Magnetic Type





Do not use Hund's rules to estimate atomic moments

Slater-Pauling plot



From: Coey

↪ Moment per atom + lattice spacing → Magnetization
 ↪ Magnetization is bounded in practice



Material	T_C (K)	M_s (kA/m)	$\mu_0 M_s$ (T)
Fe	1043	1730	2.174
Co	1394	1420	1.784
Ni	631	490	0.616
Fe ₂₀ Ni ₈₀ (Permalloy)	850	835	1.050
Fe ₃ O ₄	858	480	0.603
BaFe ₁₂ O ₁₉	723	382	0.480
Nd ₂ Fe ₁₄ B	585	1280	1.608
SmCo ₅	995	907	1.140
Sm ₂ Co ₁₇	1190	995	1.250
FePt L ₁₀	750	1140	1.433
CoPt L ₁₀	840	796	1.000
Co ₃ Pt	1100	1100	1.382



Definitions

SI system

Meter m
Kilogram kg
Second s
Ampere A

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ SI}$$

cgs-Gauss

Centimeter cm
Gram g
Second s
Ab-Ampere ab-A = 10A

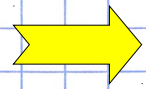
$$\mathbf{B} = \mathbf{H} + 4\pi \mathbf{M}$$

$$\mu_0 = 4\pi$$

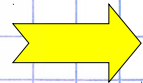
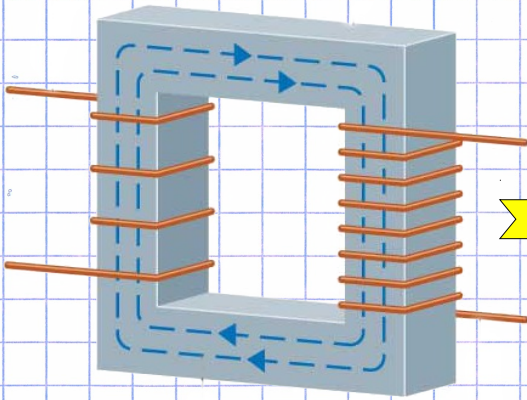
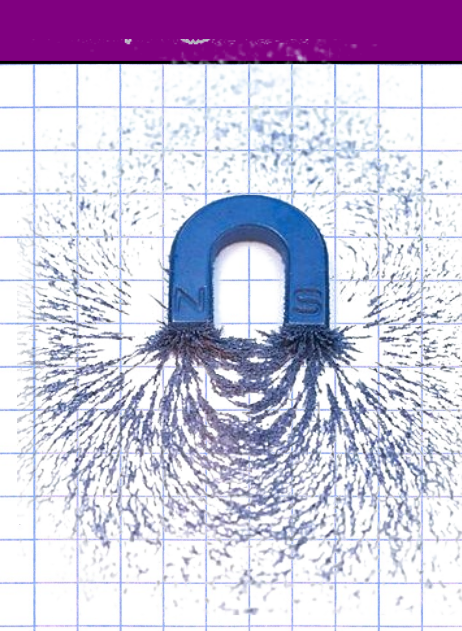
Conversions

Field	$\mathbf{H}$	1 A/m	↔	$4\pi \times 10^{-3}$ Oe (Oersted)
Moment	μ	1 A.m ²	↔	10^3 emu
Magnetization	$\mathbf{M}$	1 A/m	↔	10^{-3} emu/cm ³
Induction	$\mathbf{B}$	1 T	↔	10^4 G (Gauss)
Susceptibility	$\chi = M/H$	1	↔	$1/4\pi$

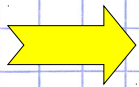
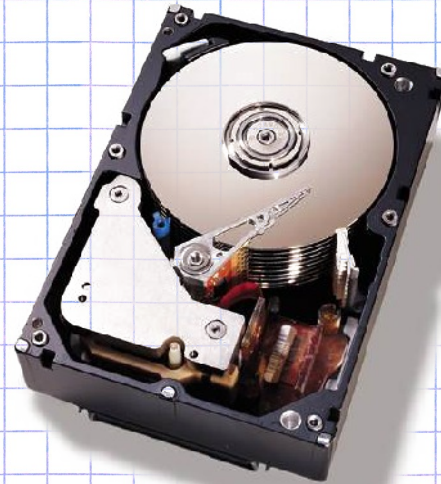
See practical on units : <http://magnetism.eu/esm/2013/abs/fruchart-tutorial.pdf>



1. Fields, moments, materials



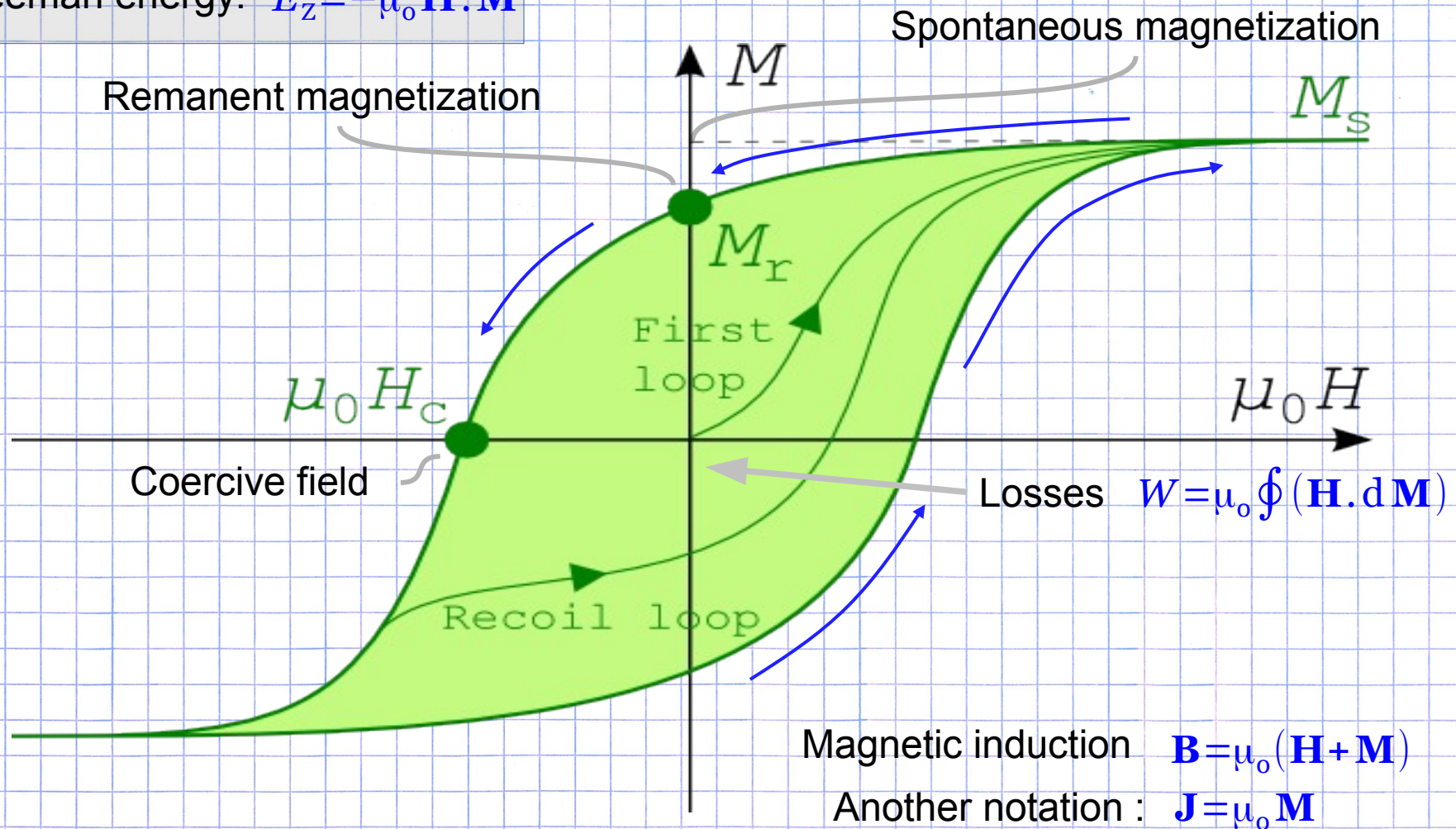
2. Magnetization processes



3. Thin film effects

Manipulation of magnetic materials:
→ Application of a magnetic field

Zeeman energy: $E_Z = -\mu_0 \mathbf{H} \cdot \mathbf{M}$



Soft magnetic materials

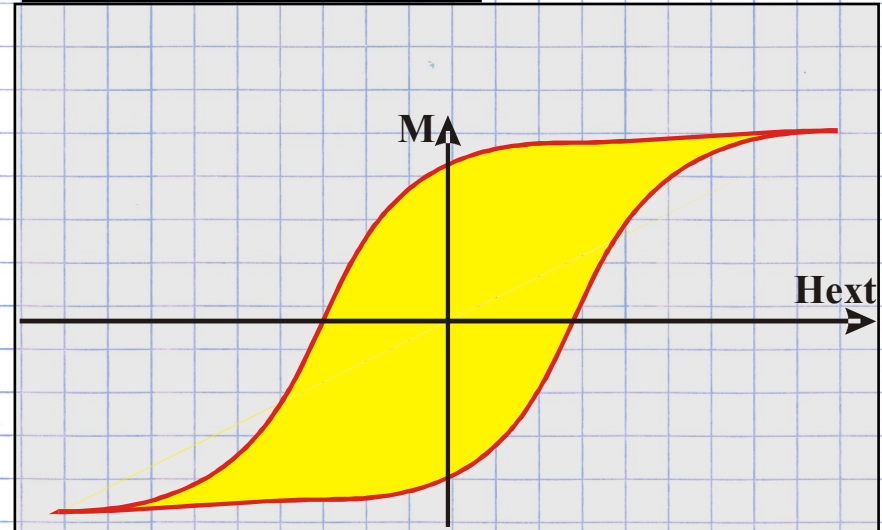


Transformers

Flux guides, sensors

Magnetic shielding

Hard magnetic materials

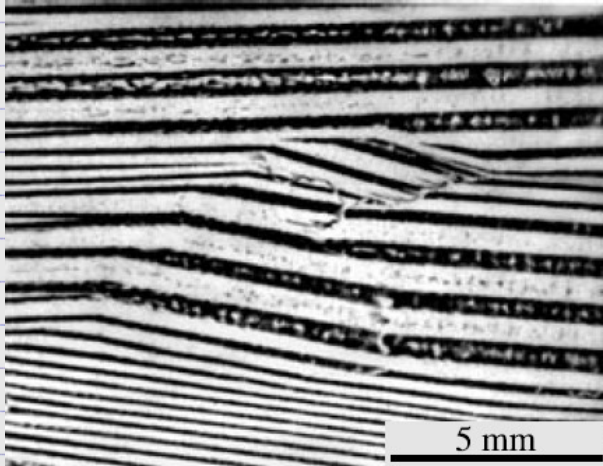


Permanent magnets, motors

Magnetic recording

Bulk material

Numerous and complex magnetic domains

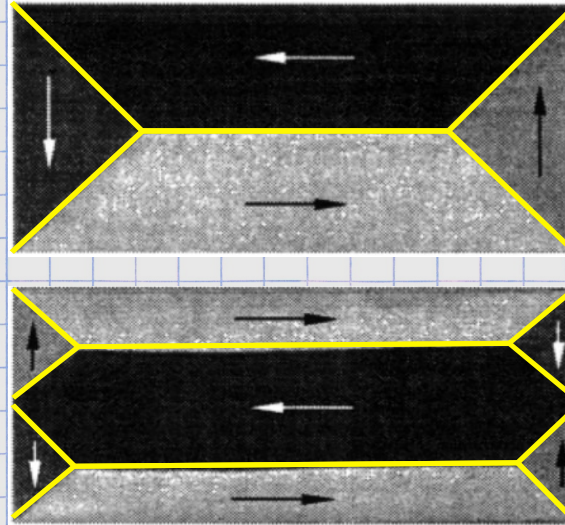


FeSi soft sheet

A. Hubert, *Magnetic domains*

Mesoscopic scale

Small number of domains, simple shape

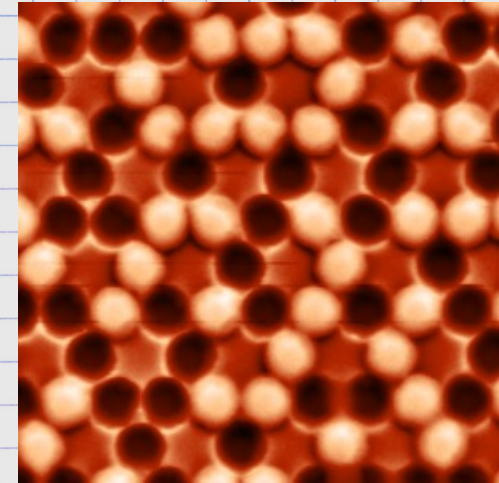


Microfabricated dots
Kerr magnetic imaging

A. Hubert, *Magnetic domains*

Nanometric scale

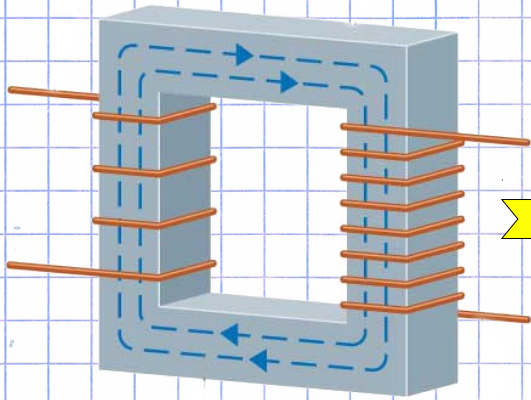
Magnetic single-domain



Nanofabricated dots
MFM

Sample courtesy :
N. Rougemaille, I. Chioar

↪ Nanomagnetism ~ mesoscopic magnetism



II. MAGNETIZATION PROCESSES

1. Magnetostatics

2. Macrospin approximation

3. Beyond macrospins



Magnetization

Magnetization vector **M** $\longrightarrow$

$$\mathbf{M} = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = M_s \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix}$$

May vary over time and space

Modulus is constant $\longrightarrow$
(hypothesis in micromagnetism)

$$m_x^2 + m_y^2 + m_z^2 = 1$$



Mean-field approach possible: $M_s = M_s(T)$

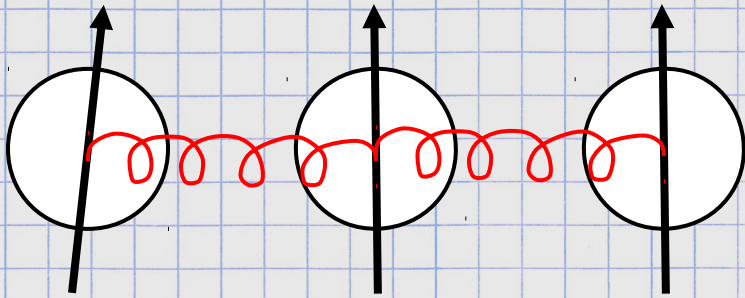
Exchange interaction

Atomistic view : $\mathcal{E} = -2 \sum_{i>j} J_{i,j} \mathbf{S}_i \cdot \mathbf{S}_j$ (total energy)

Micromagnetic view : $\mathbf{S}_i \cdot \mathbf{S}_j = S^2 \cos \theta_{i,j} \approx S^2 (1 - \theta_{i,j}^2 / 2)$

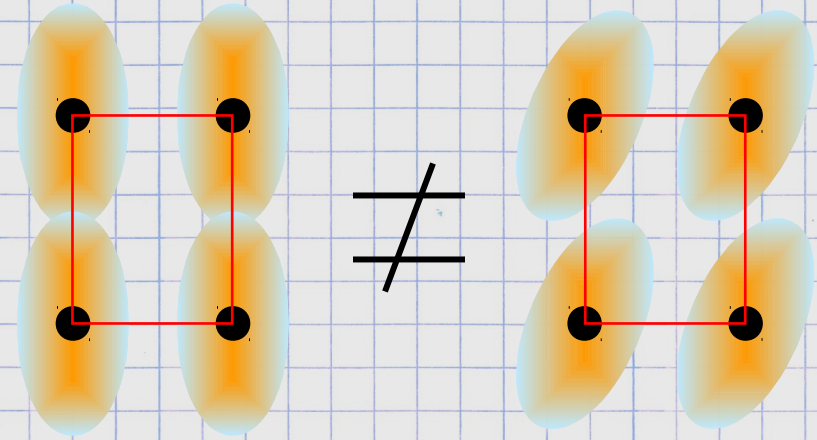
$$E = A (\nabla \cdot \mathbf{m})^2 \quad \text{(energy per unit volume)}$$

Exchange energy



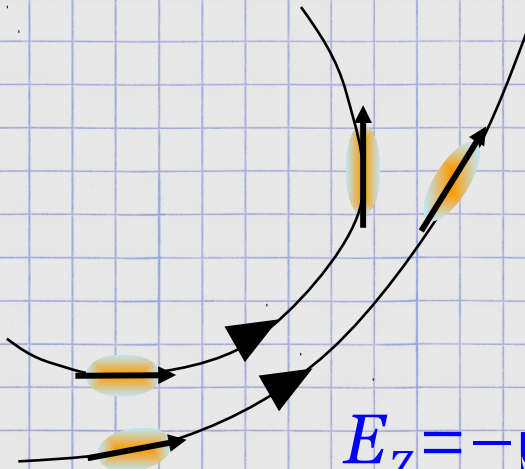
$$E_{\text{ex}} = A (\nabla \cdot \mathbf{m})^2 = A \sum_{i,j} \left(\frac{\partial m_i}{\partial x_j} \right)^2$$

Magnetocrystalline anisotropy energy



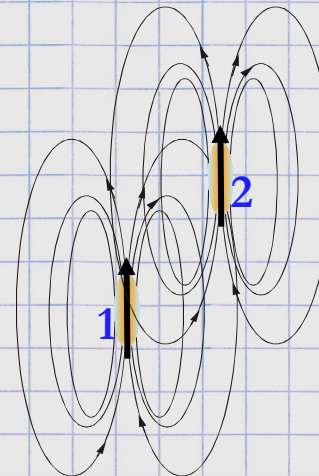
$$E_{\text{mc}} = A f(\theta, \varphi)$$

Zeeman energy



$$E_Z = -\mu_0 \mathbf{M} \cdot \mathbf{H}$$

Dipolar energy



$$E_d = -\frac{1}{2} \mu_0 \mathbf{M} \cdot \mathbf{H}_d$$



Analogy with electrostatics (synonym : magnetostatic energy) Usefull expressions

Maxwell equation $\rightarrow \text{div}(\mathbf{H}_d) = -\text{div}(\mathbf{M})$

$$\Rightarrow \mathbf{H}_d(\mathbf{r}) = -M_s \iiint_{\text{Space}} \frac{\text{div}[\mathbf{m}(\mathbf{r}')] \cdot (\mathbf{r} - \mathbf{r}')}{4\pi \|\mathbf{r} - \mathbf{r}'\|^3} dV'$$

To lift the singularity that may arise at boundaries, a volume integration around the boundaries yields:

$$\mathbf{H}_d(\mathbf{r}) = \iiint \frac{\rho(\mathbf{r}') \cdot (\mathbf{r} - \mathbf{r}')}{4\pi \|\mathbf{r} - \mathbf{r}'\|^3} dV' + \iint \frac{\sigma(\mathbf{r}') \cdot (\mathbf{r} - \mathbf{r}')}{4\pi \|\mathbf{r} - \mathbf{r}'\|^3} dS'$$

$$\mathcal{E}_d = -\frac{1}{2} \mu_0 \iiint_{\text{Sample}} \mathbf{M} \cdot \mathbf{H}_d dV$$

$$\mathcal{E}_d = \frac{1}{2} \mu_0 \iiint_{\text{Space}} \mathbf{H}_d^2 dV$$

$\Rightarrow$ Dipolar energy is always positive

$\Rightarrow$ Zero energy means a minimum

Magnetic charges

$\rho(\mathbf{r}) = -M_s \text{div}[\mathbf{m}(\mathbf{r})] \rightarrow$ volume density of magnetic charges

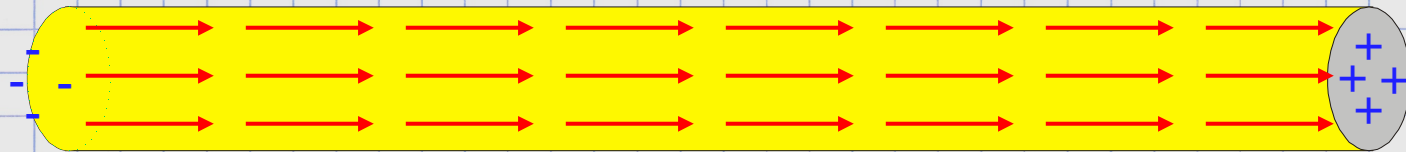
$\sigma(\mathbf{r}) = M_s \mathbf{m}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r}) \rightarrow$ surface density of magnetic charges

Notice

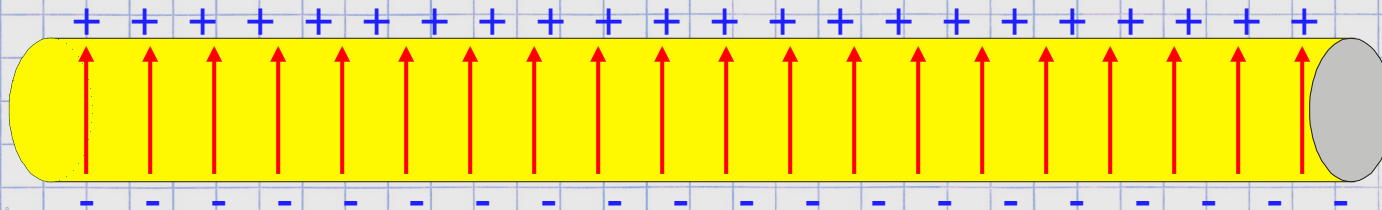
$\Rightarrow$ H_d depends on sample shape, not size

$\Rightarrow$ Synonym : dipolar $\leftrightarrow$ magnetostatic

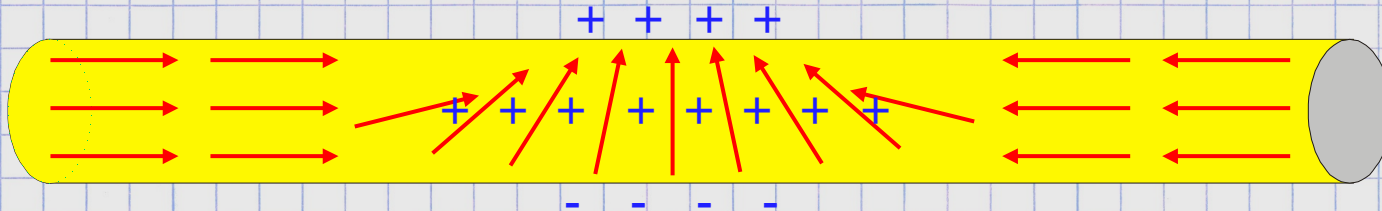
Examples of magnetic charges



Notice: no charges and $\epsilon=0$ for infinite cylinder



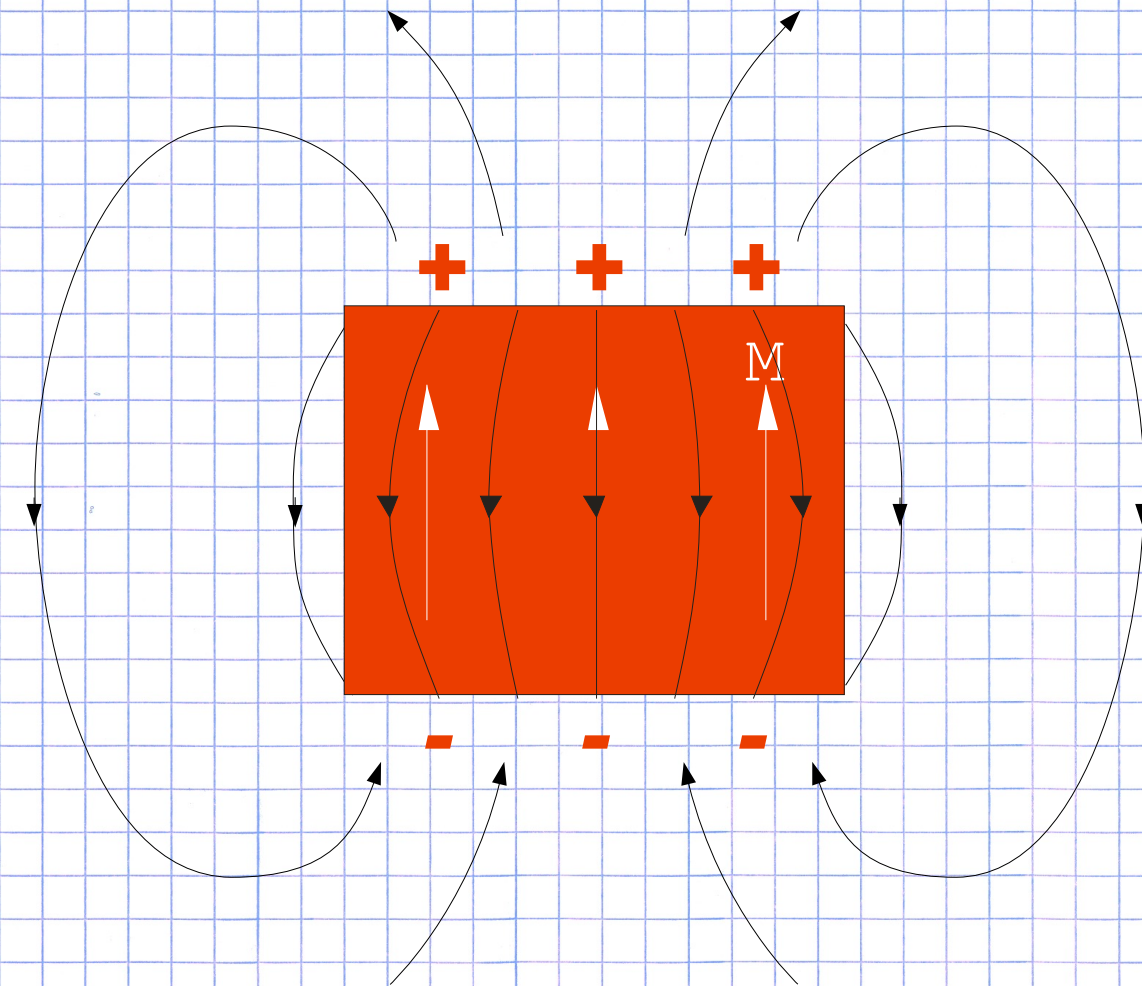
Charges on surfaces



Surface and volume charges

Take-away message

➡ Dipolar energy favors alignment of magnetization with longest direction of sample



Names

⇒ Generic names

Magnetostatic field

Dipolar field

⇒ Within sample

Demagnetizing field

⇒ Outside sample

Stray field

Principle

Goal : calculate strength of demagnetizing field and energy

Assume uniform magnetization $\mathbf{M}(\mathbf{r}) = \mathbf{M} = M_s(m_x \mathbf{u}_x + m_y \mathbf{u}_y + m_z \mathbf{u}_z)$

Demagnetizing coefficients

$$\left. \begin{aligned} \langle \mathbf{H}_d(\mathbf{r}) \rangle &= -N_i \mathbf{M} \\ \mathcal{E}_d &= N_i K_d V \end{aligned} \right\} \text{along main directions}$$

Notice: Valid for any shape

$$K_d = \frac{1}{2} \mu_0 M_s^2$$

Dipolar constant (J/m³)

$$\bar{\bar{\mathbf{N}}}$$

Demagnetizing tensor

$$N_x + N_y + N_z = 1$$



$$\begin{aligned} \langle \mathbf{H}_d \rangle_{\text{cgs}} &= -4\pi \bar{\bar{\mathbf{N}}} \cdot \mathbf{M} \\ \bar{\bar{\mathbf{D}}} &= 4\pi \bar{\bar{\mathbf{N}}} \end{aligned}$$

Model cases : slabs, cylinders and ellipsoids

(Not easy to prove, mathematics...)

H_d is uniform for surfaces of polynomial boundary with order ≤ 2

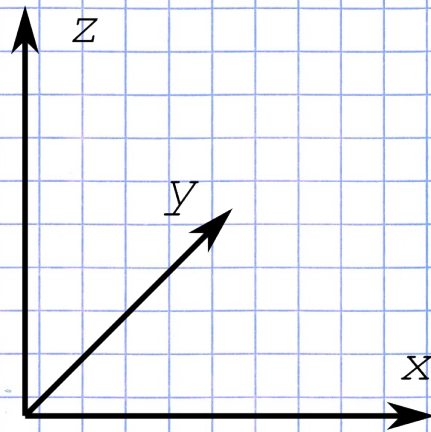
$$\mathbf{H}_d = -N_i \mathbf{M}$$

J. C. Maxwell, Clarendon 2, 66-73 (1872)

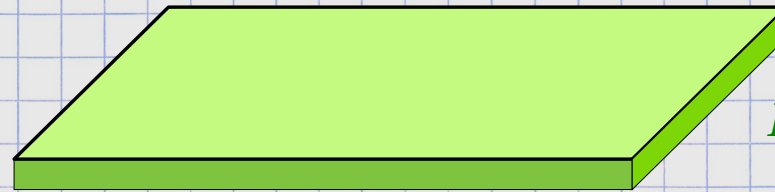
↪ Dipolar energy contributes to magnetic anisotropy with second-order term

Example :

$$\mathcal{E}_d = VK_d (N_x m_x^2 + N_y m_y^2) = VK_d (N_x - N_y) \cos^2 \theta$$



Slabs (infinite thin films)

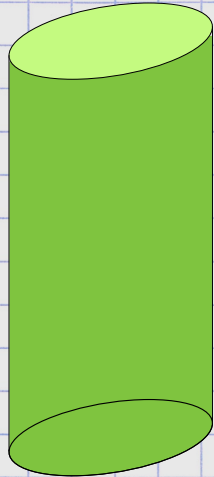


$$L_x = L_y = \infty$$

$$N_x = N_y = 0$$

$$N_z = 1$$

Cylinders (infinite)



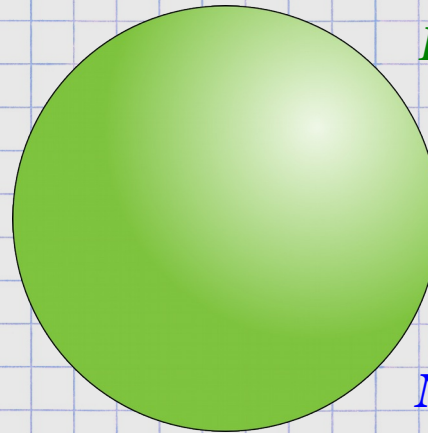
$$L_x = L_y = D$$

$$L_z = \infty$$

$$N_x = N_y = \frac{1}{2}$$

$$N_z = 0$$

Spheres



$$L_x = L_y = L_z = D$$

$$N_x = N_y = N_z = \frac{1}{3}$$



Ellipsoids

$$N_x = \frac{1}{2} abc \int_0^\infty \left[(a^2 + \eta) \sqrt{(a^2 + \eta)(b^2 + \eta)(c^2 + \eta)} \right]^{-1} d\eta \quad \text{General ellipsoid: main axes (a,b,c)}$$

$$N_x = \frac{\alpha^2}{1 - \alpha^2} \left[\frac{1}{\sqrt{1 - \alpha^2}} \operatorname{arsinh} \left(\frac{\sqrt{1 - \alpha^2}}{\alpha} \right) - 1 \right]$$

$$N_x = \frac{\alpha^2}{\alpha^2 - 1} \left[1 - \frac{1}{\sqrt{\alpha^2 - 1}} \arcsin \left(\frac{\sqrt{\alpha^2 - 1}}{\alpha} \right) \right]$$

For prolate revolution ellipsoid:
(a, c, c) with $\alpha = c/a < 1$

For oblate revolution ellipsoid:
(a, c, c) with $\alpha = c/a > 1$

$$N_y = N_z = \frac{1}{2} (1 - N_x)$$

Cylinders

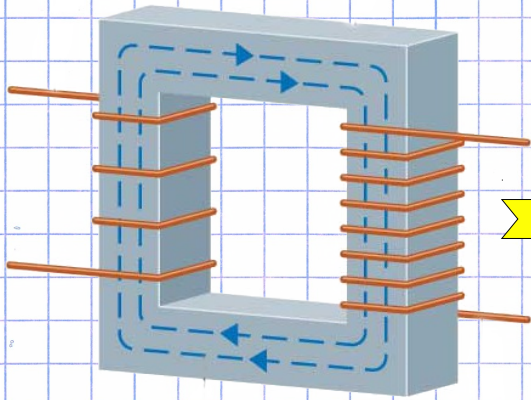
$$N_x = 0 \quad N_y = \frac{c}{b+c} \quad N_z = \frac{b}{b+c}$$

For a cylinder with axis along x

J. A. Osborn, Phys. Rev. 67, 351 (1945).

For prisms, see: **A. Aharoni, J. Appl. Phys. 83, 3432 (1998)**

More general forms, FFT approach: **M. Beleggia et al., J. Magn. Magn. Mater. 263, L1-9 (2003)**



II. MAGNETIZATION PROCESSES

1. Magnetostatics

2. Macrospin approximation

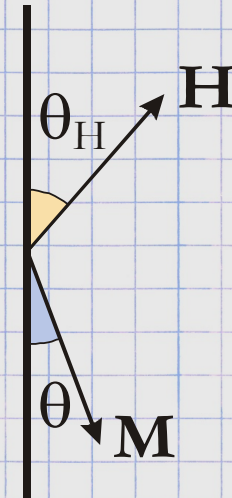
3. Beyond macrospins

Framework

Approximation: $\partial_r \mathbf{m} = \mathbf{0}$ (uniform magnetization)
(strong!)

$$\mathcal{E} = EV = V [K_{\text{eff}} \sin^2 \theta - \mu_0 M_s H \cos(\theta - \theta_H)]$$

$$K_{\text{eff}} = K_{\text{mc}} + (\Delta N) K_d$$



Dimensionless units:

$$e = \sin^2 \theta - 2h \cos(\theta - \theta_H)$$

$$e = \mathcal{E} / KV$$

$$h = H / H_a$$

$$H_a = 2K / \mu_0 M_s$$

L. Néel, *Compte rendu Acad. Sciences* 224, 1550 (1947)

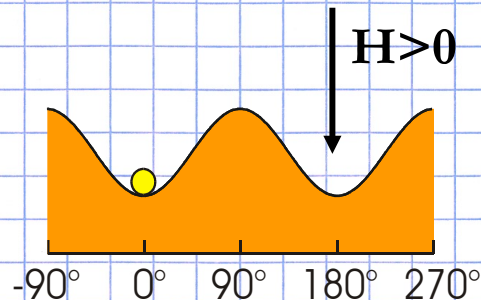
E. C. Stoner and E. P. Wohlfarth, *Phil. Trans. Royal. Soc. London* A240, 599 (1948)

IEEE Trans. Magn. 27(4), 3469 (1991) : reprint

Names used

- ↻ Uniform rotation / magnetization reversal
- ↻ Coherent rotation / magnetization reversal
- ↻ Macrospin etc.

Example for $\theta_H = 180^\circ \longrightarrow e = \sin^2 \theta + 2h \cos \theta$



Equilibrium states

$$\partial_\theta e = 2 \sin \theta (\cos \theta - h)$$

$$\partial_\theta e = 0 \implies$$

$$\cos \theta_m = h$$

$$\theta \equiv 0 [\pi]$$

Stability

$$\partial_{\theta\theta} e = 2 \cos 2\theta - 2h \cos \theta$$

$$= 4 \cos^2 \theta - 2 - 2h \cos \theta$$

$$\partial_{\theta\theta} e(0) = 2(1-h)$$

$$\partial_{\theta\theta} e(\theta_m) = 2(h^2 - 1)$$

$$\partial_{\theta\theta} e(\pi) = 2(1+h)$$

Energy barrier

$$\Delta e = e(\theta_{\max}) - e(0)$$

$$= 1 - h^2 + 2h^2 - 2h$$

$$= (1-h)^2$$



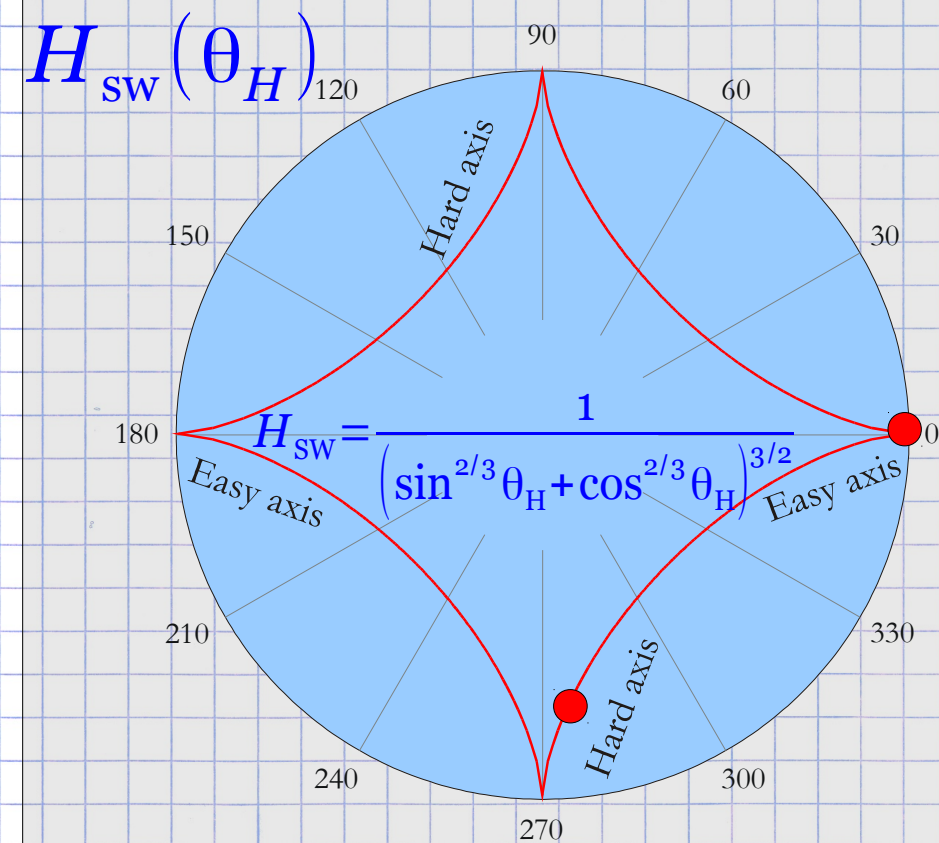
Switching

$$h = 1$$

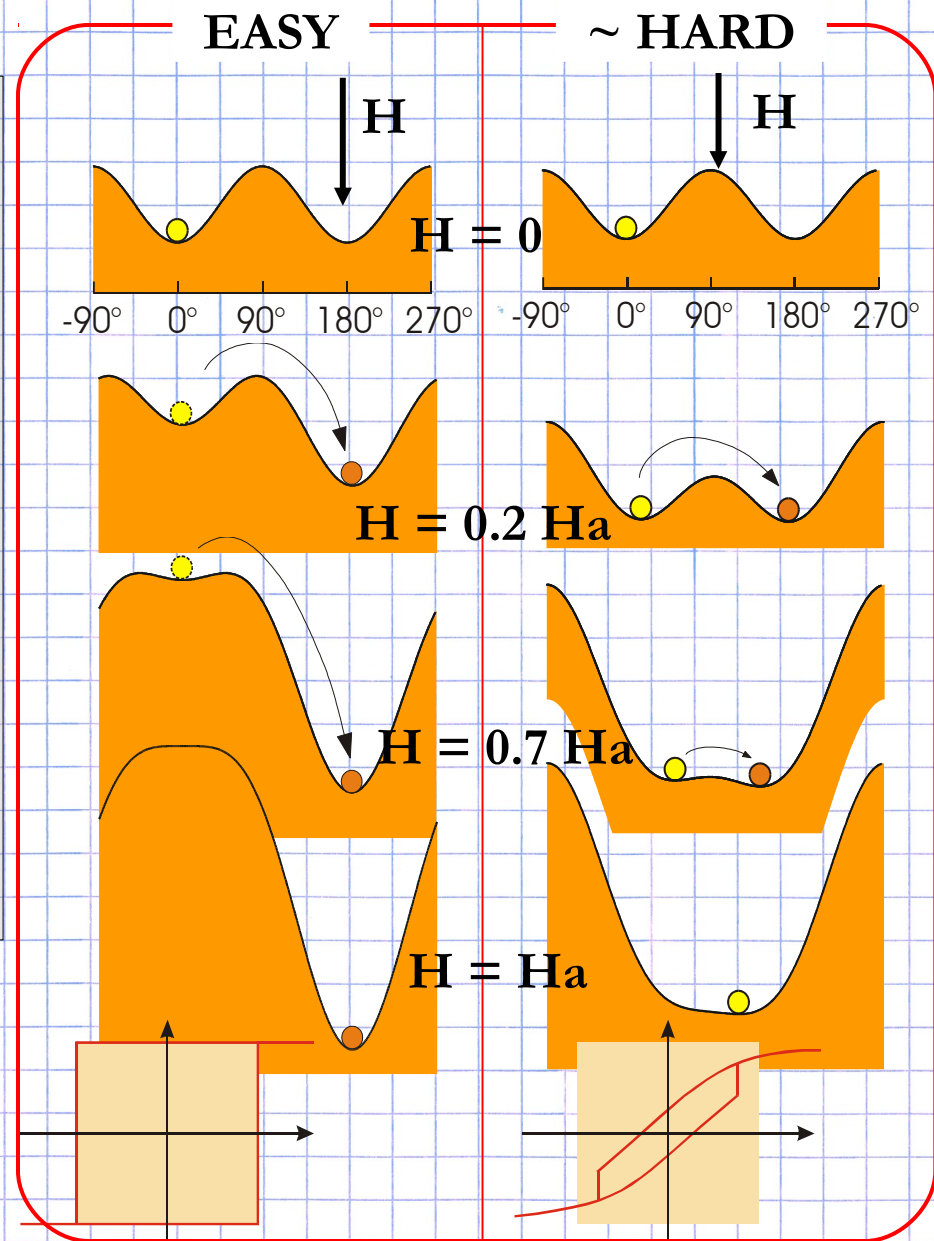
$$H = H_a = 2K / \mu_0 M_s$$

$(1-h)^\alpha$ with exponent 1.5 in general

'Astroid' curve



➤ $H_{sw}(\theta)$ is a one signature of reversal modes



J. C. Slonczewski, Research Memo RM
003.111.224, IBM Research Center (1956)



Switching field = Reversal field

A value of field at which an irreversible (abrupt) jump of magnetization angle occurs.

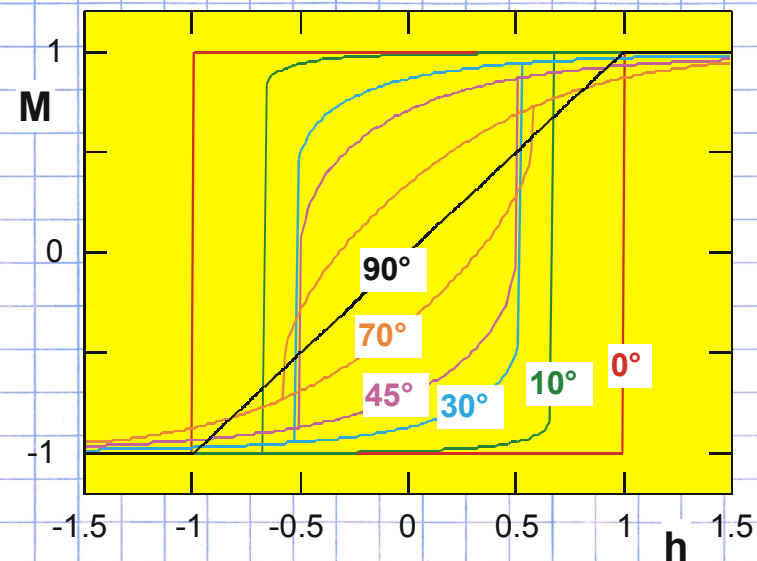
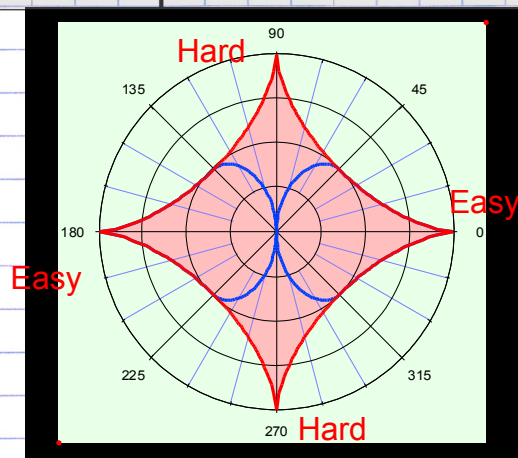
Can be measured only in single particles.

Coercive field

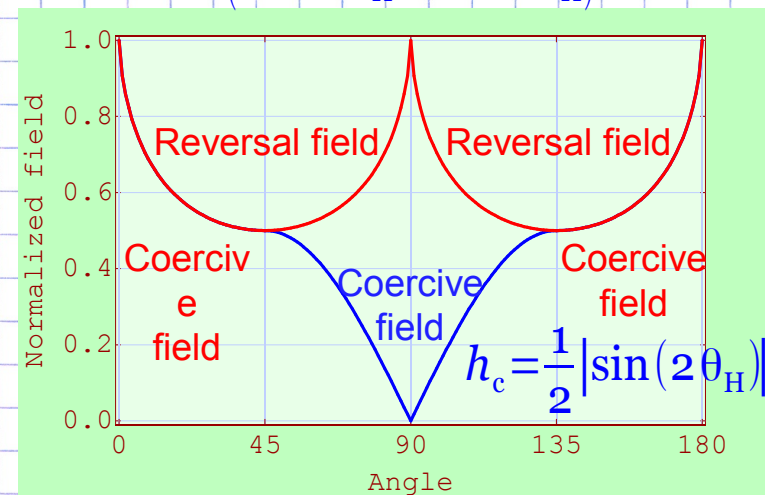
The value of field at which $\mathbf{M} \cdot \mathbf{H} = 0$ ($\theta = \theta_H \pm \pi/2$)

A quantity that can be measured in real materials (large number of 'particles').

May be or may not be a measure of the mean switching field at the microscopic level



$$H_{sw} = \frac{1}{(\sin^{2/3} \theta_H + \cos^{2/3} \theta_H)^{3/2}}$$

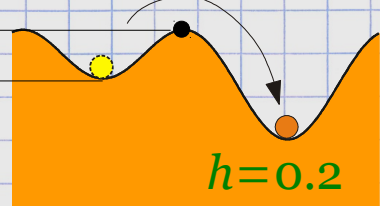


Barrier height

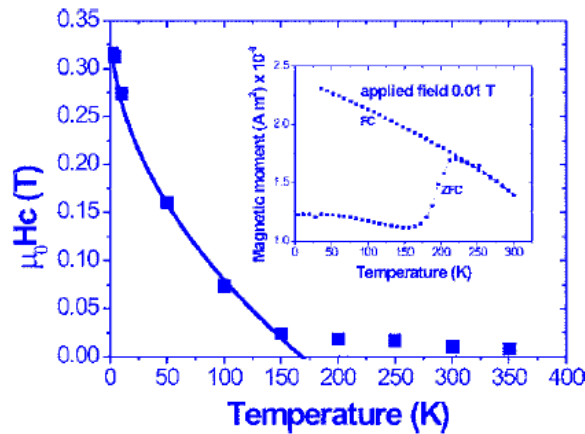
$$\Delta e = e(\theta_{\max}) - e(0) = (1-h)^2$$

 Δe

$$(h = \mu_0 M_s H / 2K)$$

 $h = 0.2$


J. Appl. Phys. **99**, 08Q514 (2006)



Thermal activation

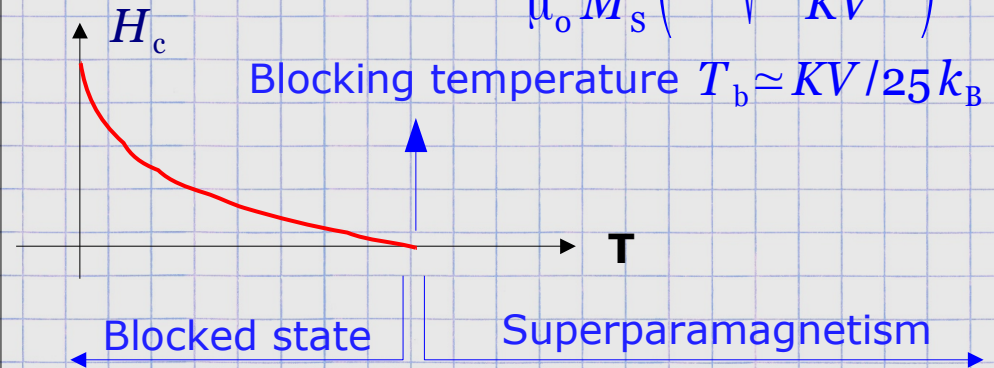
Brown, Phys.Rev.130, 1677 (1963)

$$\tau = \tau_0 \exp\left(\frac{\Delta \mathcal{E}}{k_B T}\right) \Rightarrow \Delta \mathcal{E} = k_B T \ln(\tau / \tau_0)$$

$$\tau_0 \approx 10^{-10} \text{ s}$$

Lab measurement: $\tau \approx 1 \text{ s} \Rightarrow \Delta \mathcal{E} \approx 25 k_B T$

$$\Rightarrow H_c = \frac{2K}{\mu_0 M_s} \left(1 - \sqrt{\frac{25 k_B T}{KV}}\right)$$



E. F. Kneller, J. Wijn (ed.) Handbuch der Physik XIII/2: Ferromagnetismus, Springer, 438 (1966)

M. P. Sharrock, J. Appl. Phys. **76**, 6413-6418 (1994)

Notice, for magnetic recording : $\tau \approx 10^9 \text{ s}$ $KV_b \approx 40 - 60 k_B T$



Formalism

C. P. Bean & J. D. Livingston, J. Appl. Phys. 30, S120 (1959)

Energy

$$E = KV f(\theta, \varphi) - \mu_0 \mu H$$

Partition function

$$Z = \sum \exp(-\beta E)$$

Average moment

$$\langle \mu \rangle = \frac{1}{\beta \mu_0 Z} \frac{\partial Z}{\partial H}$$

Isotropic case

$$Z = \int_{-\mathcal{N}}^{\mathcal{N}} \exp(-\beta E) d\mu$$

Note: equivalent to integration over solid angle

$$\langle \mu \rangle = \mathcal{N} \left[\coth\left(x - \frac{1}{x}\right) \right]$$

Langevin function

Note:

Use the moment M of the particule, not spin $\frac{1}{2}$.

$$x = \beta \mu_0 \mathcal{N} H$$

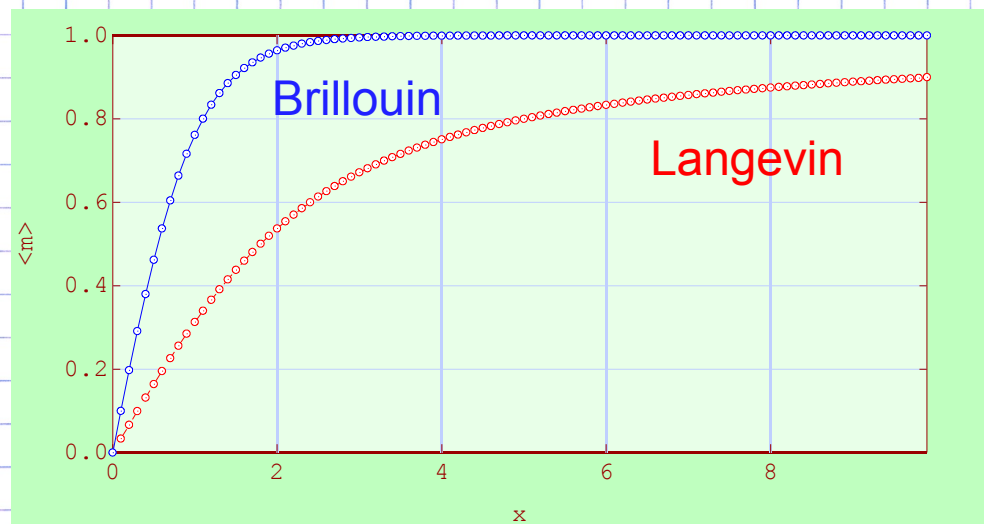


Infinite anisotropy

$$Z = \exp(\beta \mu_0 \mathcal{N} H) + \exp(-\beta \mu_0 \mathcal{N} H)$$

$$\langle \mu \rangle = \mathcal{N} \tanh(x)$$

Brillouin $\frac{1}{2}$ function



Material	T_C (K)	M_s (kA/m)	$\mu_0 M_s$ (T)	K (kJ/m ³)
Fe	1043	1730	2.174	48
Co	1394	1420	1.784	530
Ni	631	490	0.616	-4.5
Fe ₂₀ Ni ₈₀ (Permalloy)	850	835	1.050	≈ 0
Fe ₃ O ₄	858	480	0.603	-13
BaFe ₁₂ O ₁₉	723	382	0.480	250
Nd ₂ Fe ₁₄ B	585	1280	1.608	4900
SmCo ₅	995	907	1.140	17000
Sm ₂ Co ₁₇	1190	995	1.250	3300
FePt L ₁₀	750	1140	1.433	6600
CoPt L ₁₀	840	796	1.000	4900
Co ₃ Pt	1100	1100	1.382	2000

↪ MRAM : anisotropy comes from shape or interfaces (see later) → materials with high anisotropy not wanted → Fe₂₀Ni₈₀, CoFe(B) etc. Avoid pure Co, Fe (bcc)

↪ Other effects not discussed : magneto-elastic anisotropy → avoid pure Ni

Basics of precessional switching

Demonstration: 1999

Magnetization dynamics:

Landau-Lifshitz-Gilbert equation:

$$\frac{d\mathbf{m}}{dt} = -|\gamma_0| \mathbf{m} \times \mathbf{H} + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt}$$

γ_0 Gyromagnetic ratio

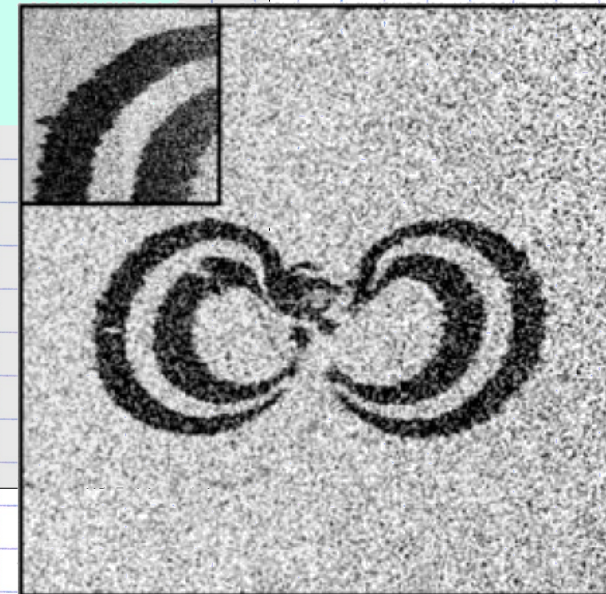
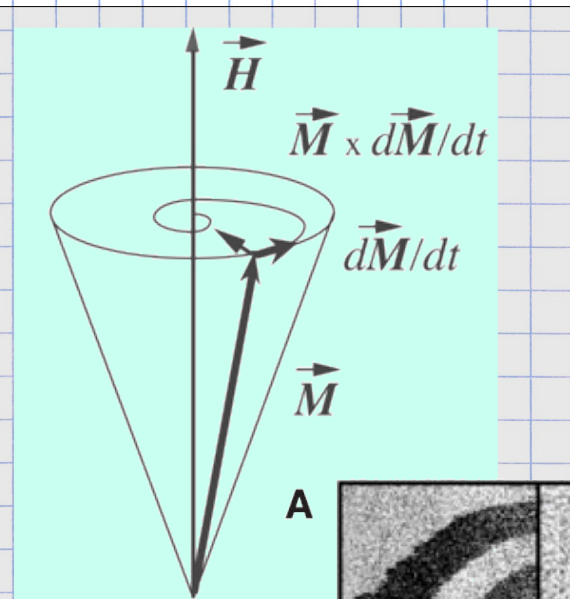
$$\gamma_0 = \mu_0 \gamma \quad \gamma = -g_J \frac{e}{2m_e} < 0$$

$$\gamma_{\text{spin}} / 2\pi \approx 28 \text{ GHz/T}$$

H_{eff} Effective field
(including applied)

$$H_{\text{eff}} = -\frac{1}{\mu_0 M_s} \frac{\partial E_{\text{mag}}}{\partial \mathbf{m}}$$

α Damping coefficient $\alpha = 10^{-1} - 10^{-3}$

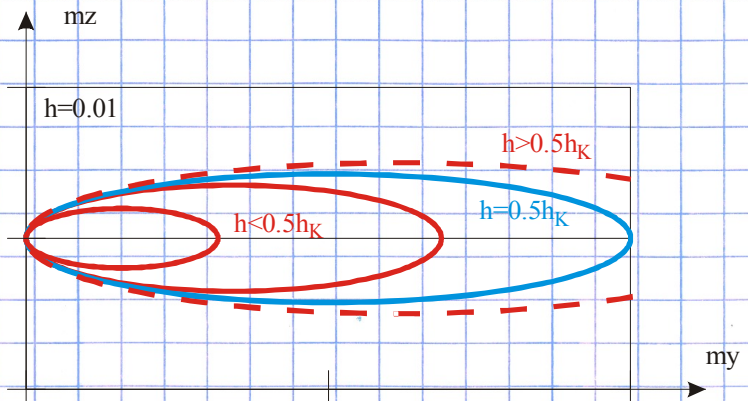
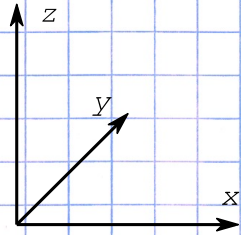
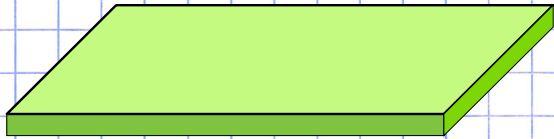


150 μm

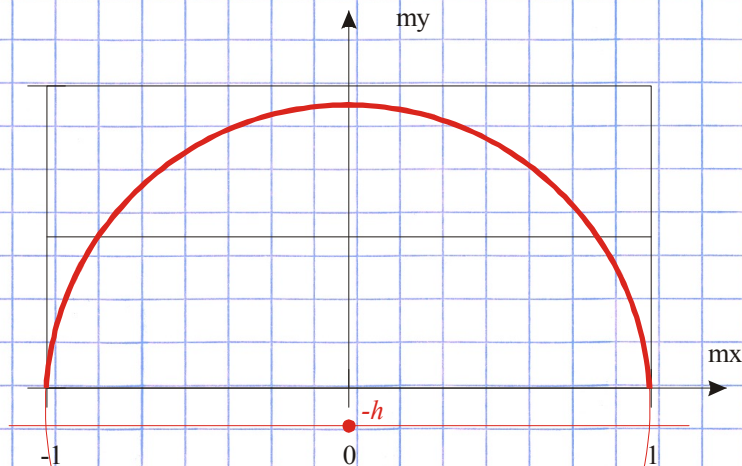
C. Back et al., Science 285, 864 (1999)



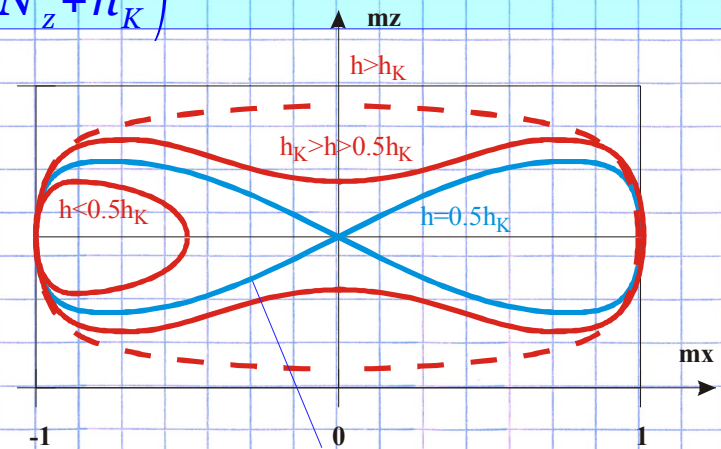
Magnetization trajectories



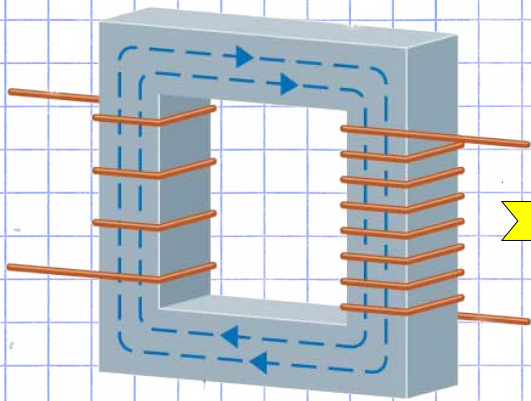
$$\frac{\left(\frac{m_z^2}{\frac{h_K}{N_z + h_K}}\right)}{\left(\frac{h_K}{N_z + h_K}\right)} + \left(m_y - \frac{h}{h_K}\right)^2 = \left(\frac{h}{h_K}\right)^2$$



$$m_x^2 + \frac{(m_y + h/N_z)^2}{1 + h_K/N_z} = 1 + \frac{h^2}{N_z(N_z + h_K)}$$



$$\omega \approx 0.847 \gamma_o \sqrt{M_S(H - H_K)/2}$$



II. MAGNETIZATION PROCESSES

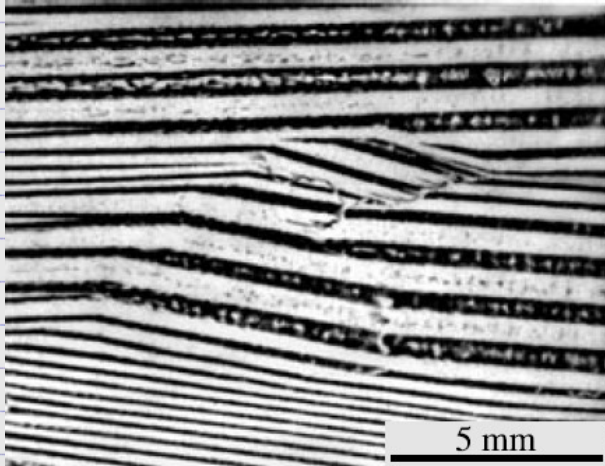
1. Magnetostatics

2. Macrospin approximation

3. Beyond macrospins

Bulk material

Numerous and complex magnetic domains

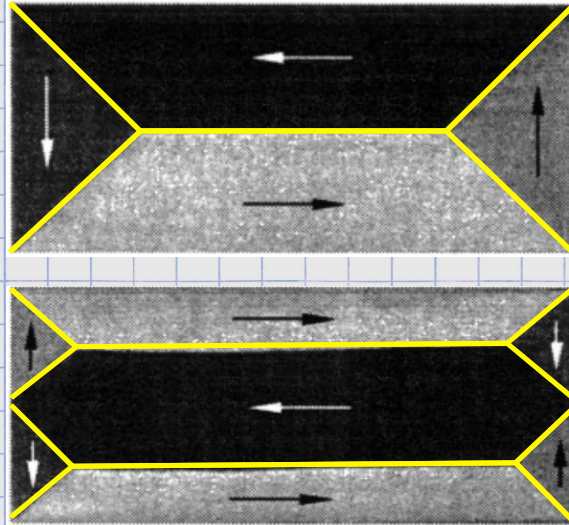


FeSi soft sheet

A. Hubert, Magnetic domains

Mesoscopic scale

Small number of domains, simple shape

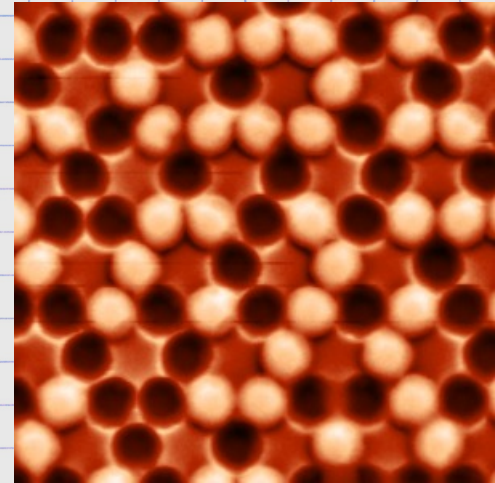


Microfabricated dots
Kerr magnetic imaging

A. Hubert, Magnetic domains

Nanometric scale

Magnetic single-domain



Nanofabricated dots
MFM

*Sample courtesy :
N. Rougemaille, I. Chioar*



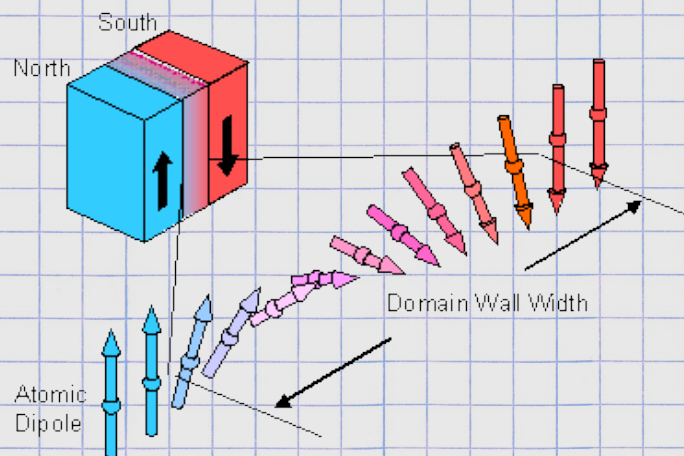
Anisotropy exchange length

$$E = A(\partial_x \theta)^2 + K \sin^2 \theta$$

Exchange $\rightarrow$ J/m Anisotropy $\rightarrow$ J/m³

Anisotropy exchange length: $\Delta_u = \sqrt{A/K}$

$\Delta_u \approx 1 \text{ nm}$ → $\Delta_u \geq 100 \text{ nm}$
Hard Soft



Often called *Bloch parameter*
or *domain-wall width*

Dipolar exchange length

$$E = A(\partial_x \theta)^2 + K_d \sin^2 \theta$$

Exchange $\rightarrow$ J/m Dipolar energy $\rightarrow$ J/m³

$$K_d = \frac{1}{2} \mu_0 M_s^2$$

Dipolar exchange length:

$$\Delta_d = \sqrt{A/K_d}$$

$$= \sqrt{2 A / \mu_0 M_s^2}$$

$\Delta_d \approx 3 - 10 \text{ nm}$

Single-domain critical size
relevant for nanoparticles
made of soft magnetic material



Often called *Exchange length*

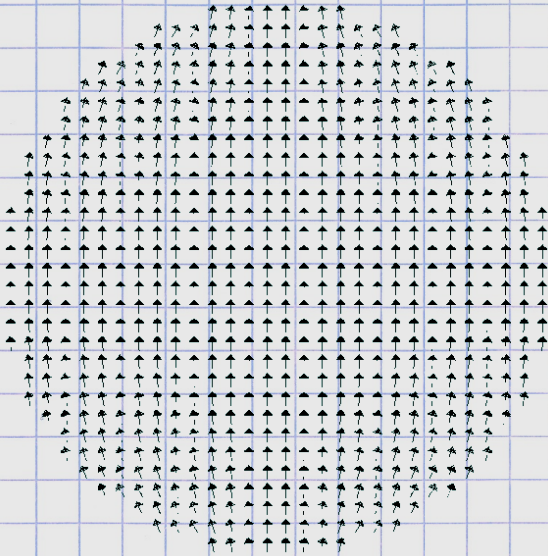
Notice:

Other length scales: with field etc.

Geometry

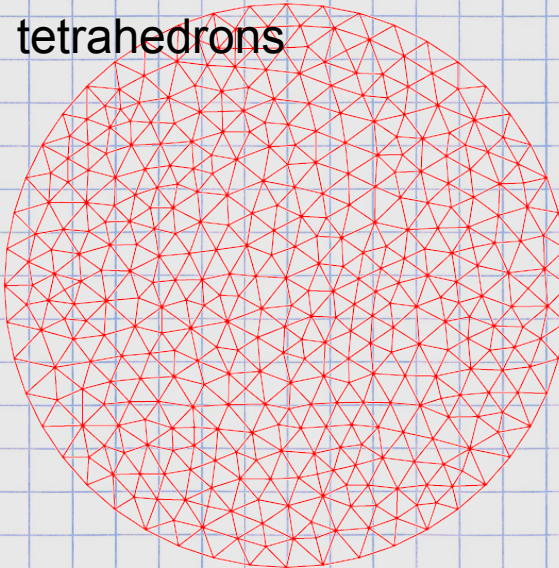
Finite differences

Cells = prisms



Finite elements

Cells = tetrahedrons



Constraints

- ⇒ Cell size smaller than magnetic length scales
- ⇒ Time step $< 1\text{ps}$
- ⇒ Limiting step : calculation of dipolar field

Features

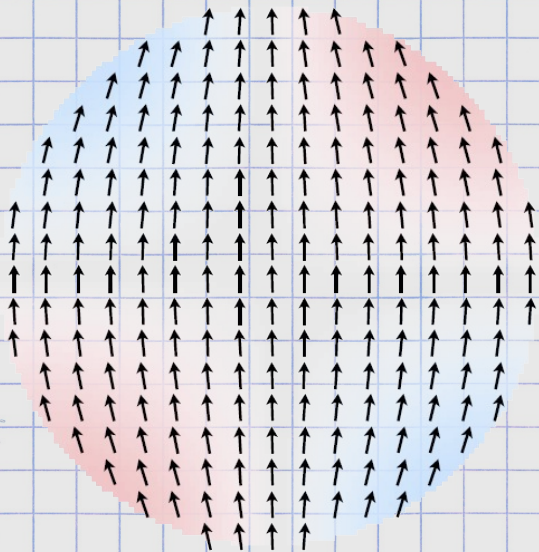
- ⇒ Map of magnetization
- ⇒ Magnetization dynamics
- ⇒ Coupling with transport : still imperfect
- ⇒ Long time & temperature : difficult

Some codes

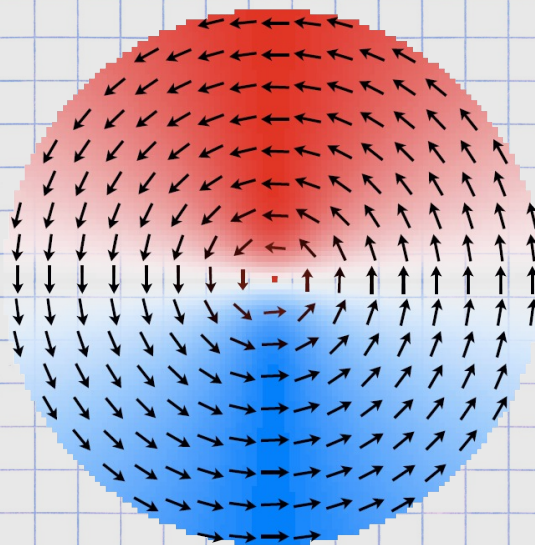
- ⇒ Proprietary
- ⇒ FD : OOMMF (soon on GPU), Mumax, Micromagus, GPMagnet ...
- ⇒ FE : LLG micromagnetics simulator, Magpar, Nmag ...

<http://magnetism.eu/links/tools.html>

Flat disk ('dot')

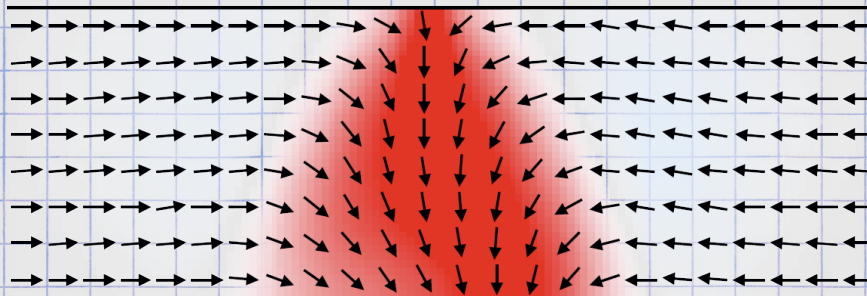


Near single-domain



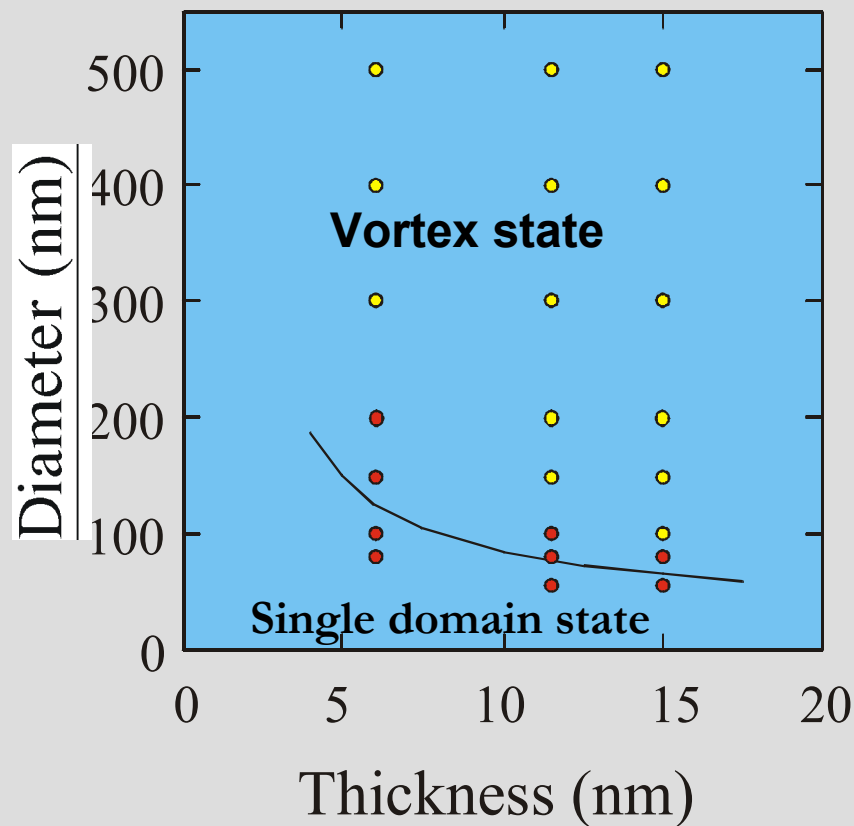
Vortex state (flux closure)

Strip (one-dimensional track)



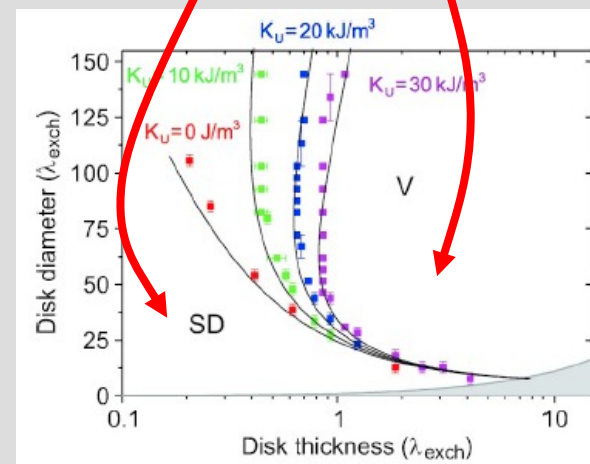
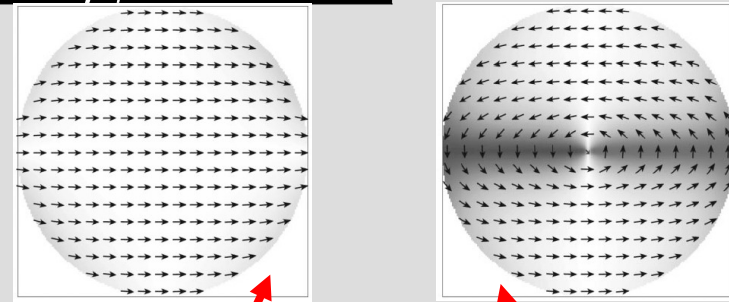
'Transverse' domain wall

Experiments



R.P. Cowburn,
J.Phys.D:Appl.Phys.33, R1-R16 (2000)

Theory / Simulation



Zero-field
cross-over

$$t D \approx 20 \Delta_d^2$$



P.-O. Jubert & R. Allenspach,
PRB 70, 144402/1-5 (2004)

- Vortex state (flux-closure) dominates at large thickness and/or diameter
- The size limit for single-domain is much larger than the exchange length
- Experimentally the vortex may be difficult to reach close to the transition (hysteresis)

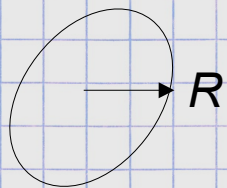
Upper bound for dipolar fields in 2D

Estimation of an upper range of dipolar field in a 2D system

$$\|\mathbf{H}_d(R)\| \leq \int_0^R \frac{2\pi r}{r^3} dr$$

Integration

Local dipole $1/r^3$

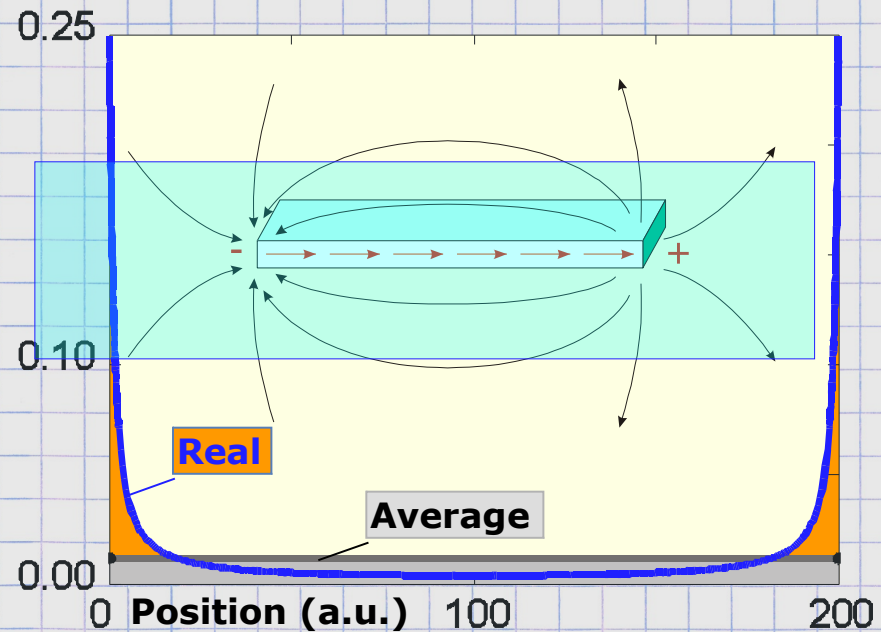


$$\|\mathbf{H}_d(R)\| = \text{Cte} + \mathcal{O}(1/R)$$

Convergence with finite radius
(typically thickness)

Non-homogeneity of dipolar fields in 2D

Example: flat strip with thickness/height = 0.0125



- ↪ Dipolar fields are weak and short-ranged in 2D or even lower-dimensionality systems
- ↪ Dipolar fields can be highly non-homogeneous in anisotropic systems like 2D
- ↪ Consequences on dot's non-homogenous state, magnetization reversal, collective effects etc.

Configurational anisotropy: deviations from single-domain

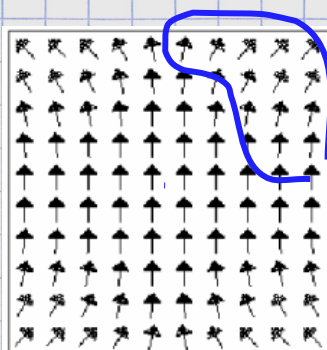
Strictly speaking, 'shape anisotropy' is of second order:

$$E_d = \frac{1}{2} \mu_0 (N_x M_x^2 + N_y M_y^2 + N_z M_z^2)$$

2D: $\mathcal{E}_d = V K_d \sin^2 \theta$

In real samples magnetization is never perfectly uniform: competition between exchange and dipolar

Num.Calc. (100nm)



Flower state
 $c/a > 2.7$



Leaf state
 $c/a < 2.7$

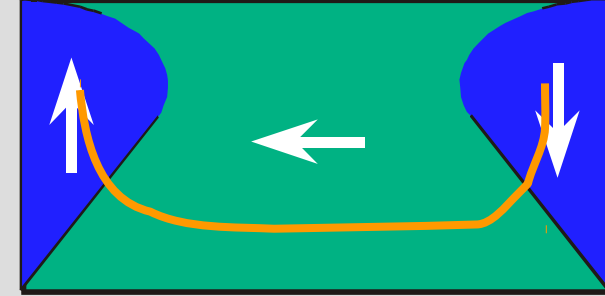
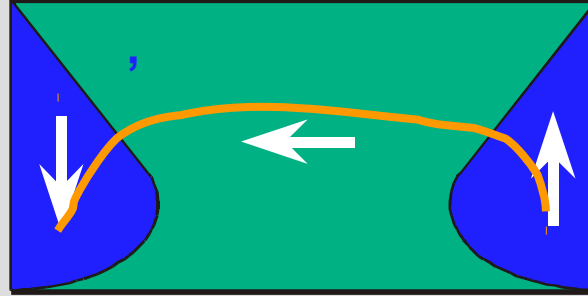
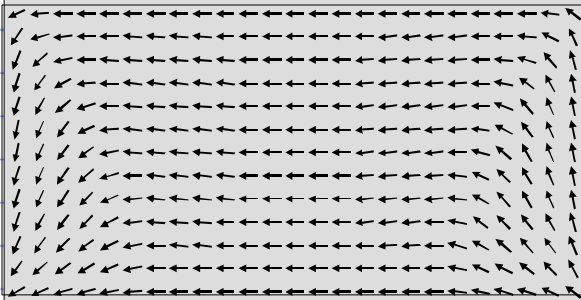
Configurational anisotropy may be used to stabilize configurations against switching

➡ Higher-order contributions to magnetic anisotropy

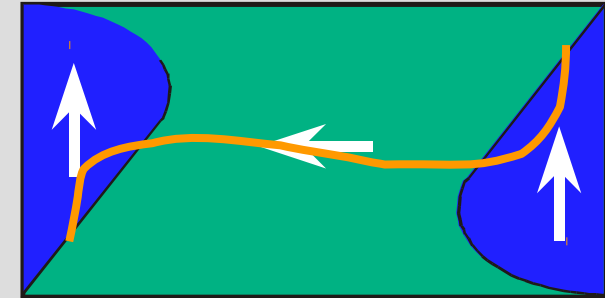
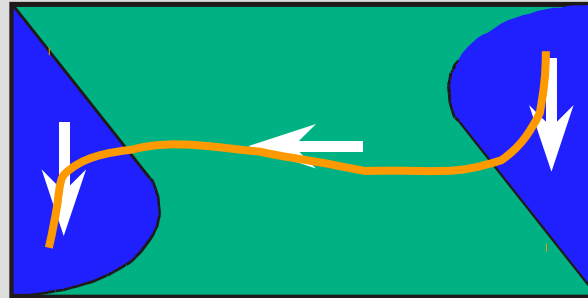
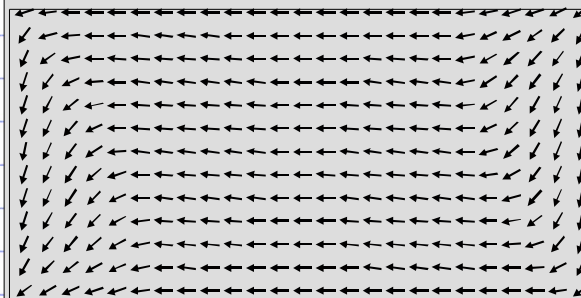
M. A. Schabes et al., JAP 64, 1347 (1988)

R.P. Cowburn et al., APL 72, 2041 (1998)

'C' state



'S' state



↪ At least 8 nearly-equivalent ground-states for a rectangular dot
 ↪ Issue for the reproducibility of magnetization reversal

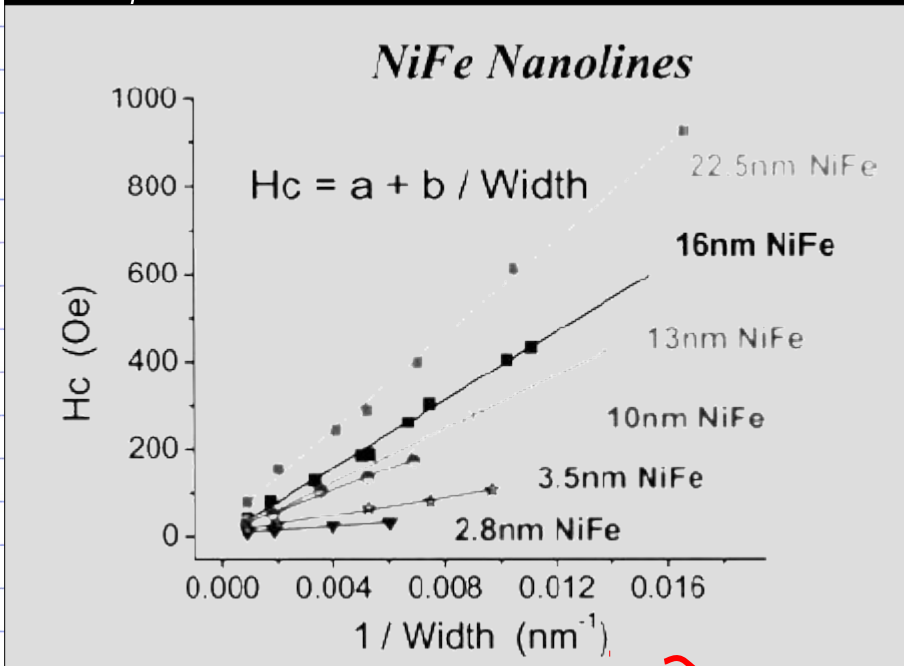


Hypotheses

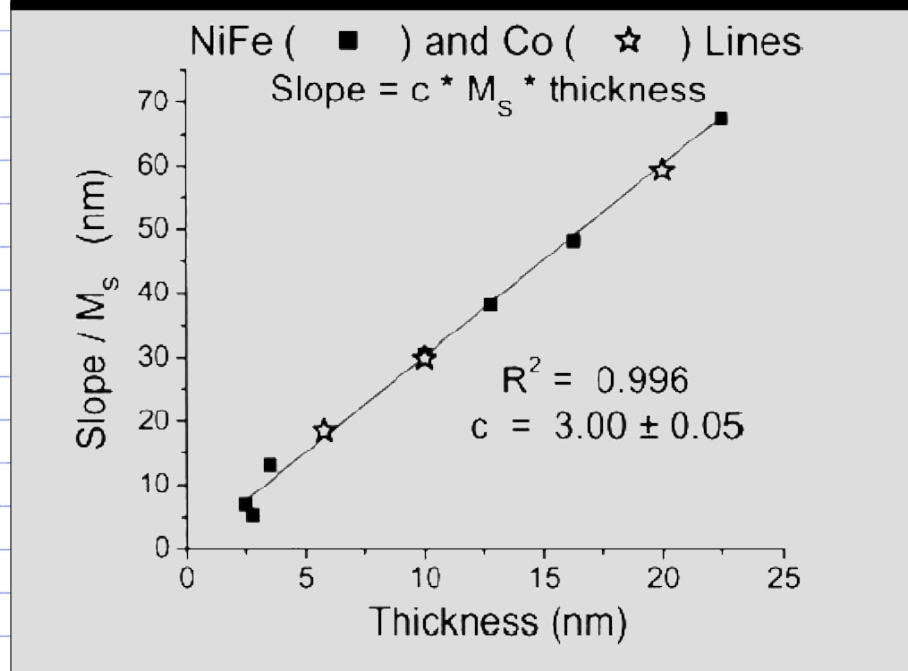
- ⇒ Soft magnetic material
- ⇒ Not too small neither too large nanostructures



$H_c \sim 1/\text{Width}$



$H_c \sim M_s * \text{Thickness}$



$$H_c \approx a + 3M_s \frac{t}{W}$$

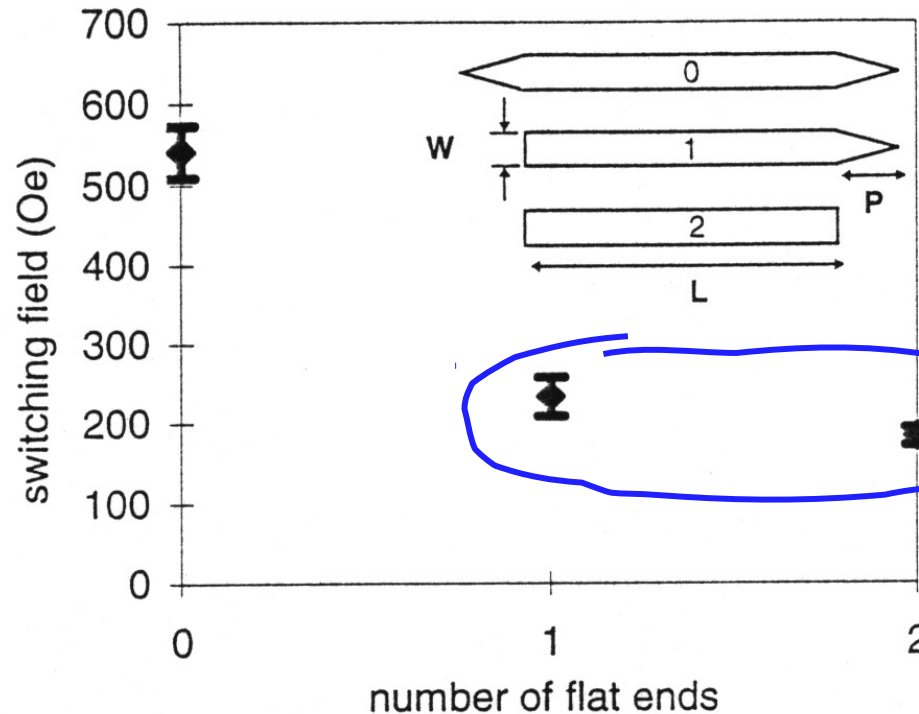
~Lateral demagnetizing coefficient of the strip



W. C. Uhlig & J. Shi,
Appl. Phys. Lett. 84, 759 (2004)

Magnetization is pinned at sharp ends

Experiments Permalloy (soft)

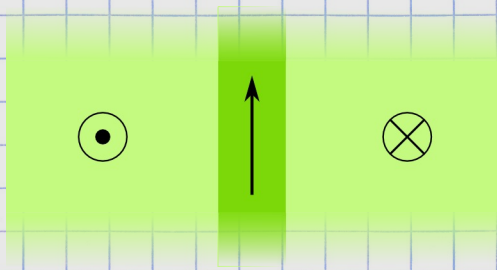


Similar

Fig. 8 Dependence of switching field of acicular elements on the number of flat ends. Element geometry is also shown: $L=2.5\mu\text{m}$, $W=200\text{ nm}$, $P=500\text{ nm}$.

K.J. Kirk et al., J. Magn. Soc. Jap., 21 (7), (1997)

Bloch domain wall in the bulk (2D)



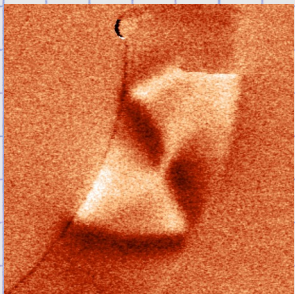
- ⇒ No magnetostatic energy
- ⇒ Width $\Delta_u = \sqrt{A/K}$
- ⇒ Areal energy $\gamma_w = 4\sqrt{AK}$



Other angles & anisotropy

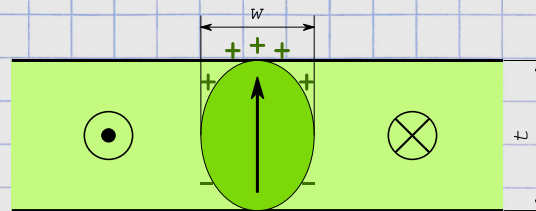
F. Bloch, Z. Phys. 74, 295 (1932)

Constrained walls (eg : in stripes)



Permalloy (15nm)
Strip 500nm

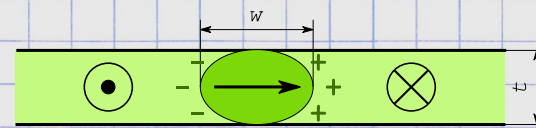
Domain walls in thin films (2D → 1D)



Bloch wall

$$t \gtrsim w$$

300x800nm



Néel wall

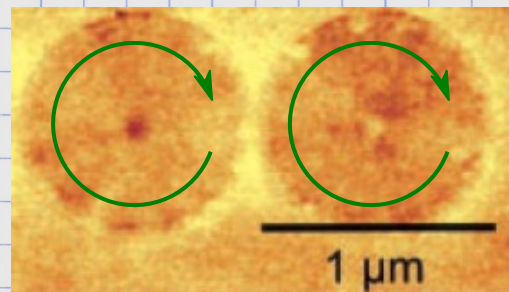
$$t \lesssim w$$

1000x2000nm

- ⇒ Contains magnetostatic energy
- ⇒ No exact analytics

L. Néel, C. R. Acad. Sciences 241, 533 (1956)

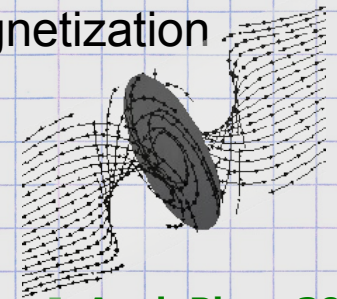
Magnetic vortex (1D → 0D)



T. Shinjo et al.,
Science 289, 930 (2000)

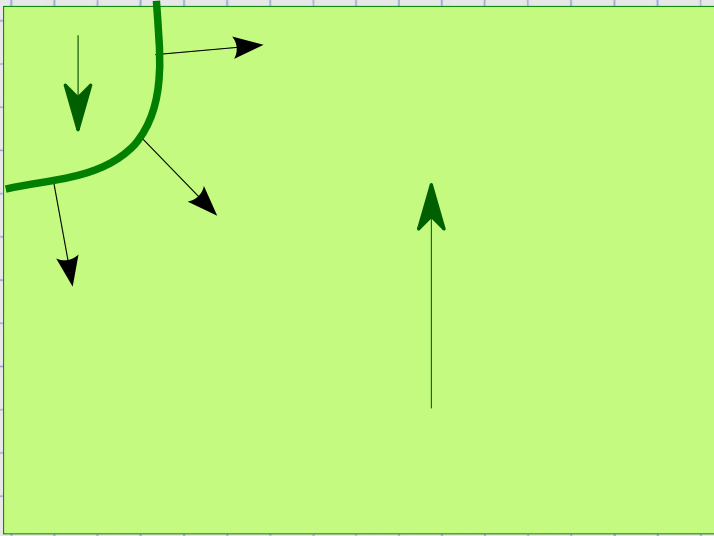
Bloch point (0D)

- ⇒ Point with vanishing magnetization



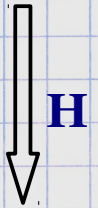
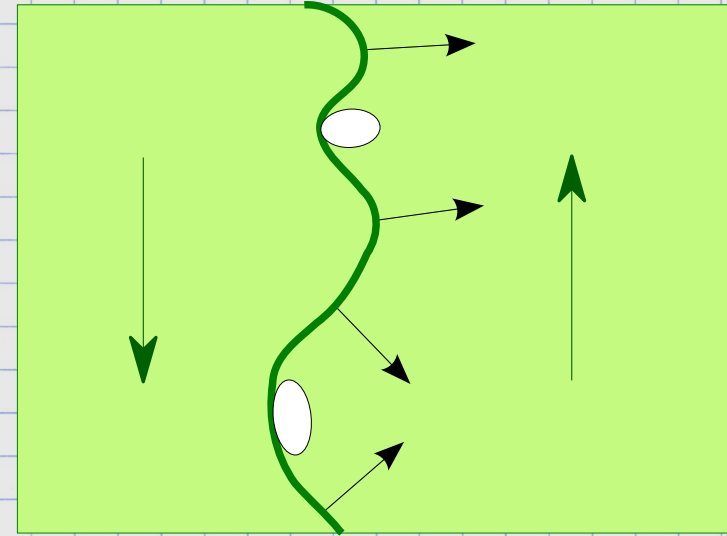
W. Döring, J. Appl. Phys. 39,
1006 (1968)

Coercivity determined by nucleation

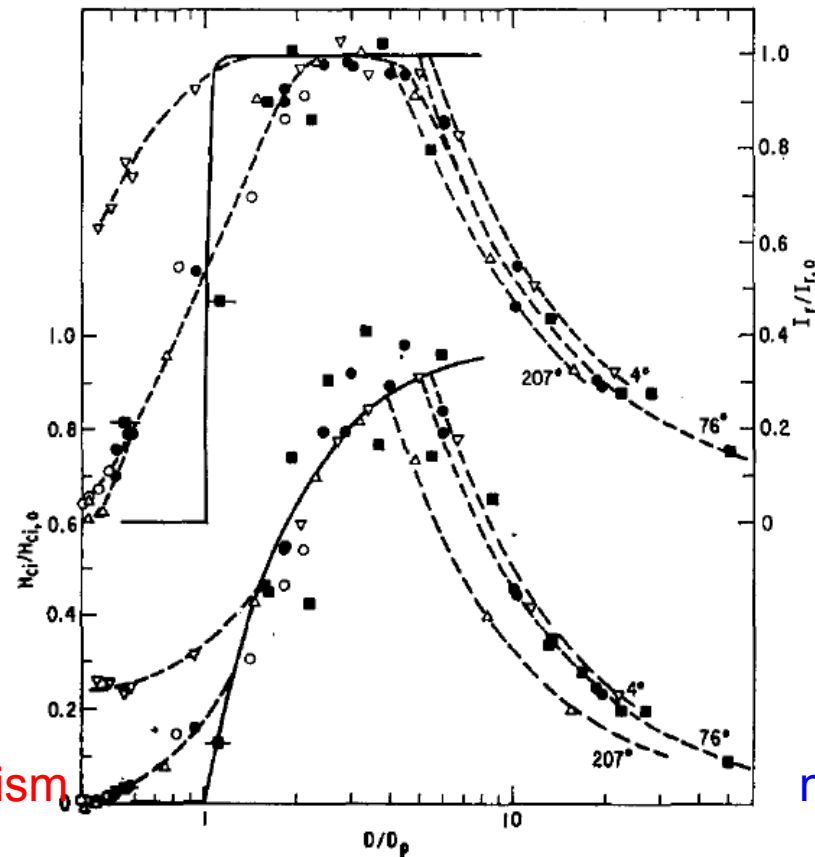


- ⇒ Physics has some similarity with that of grains
- ⇒ Concept of nucleation volume

Coercivity determined by propagation



- ⇒ Physics of surface/string in heterogeneous landscape
- ⇒ Modeling necessary

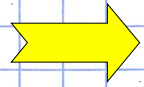


Towards
superparamagnetism

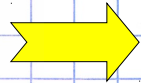
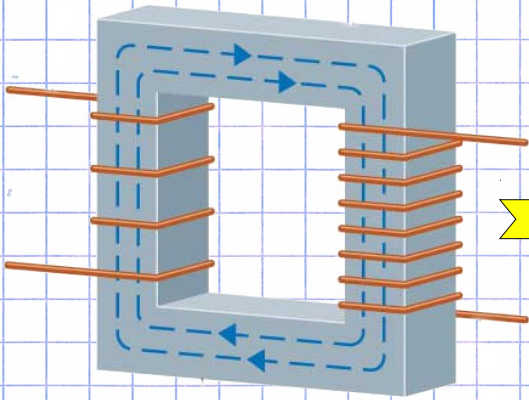
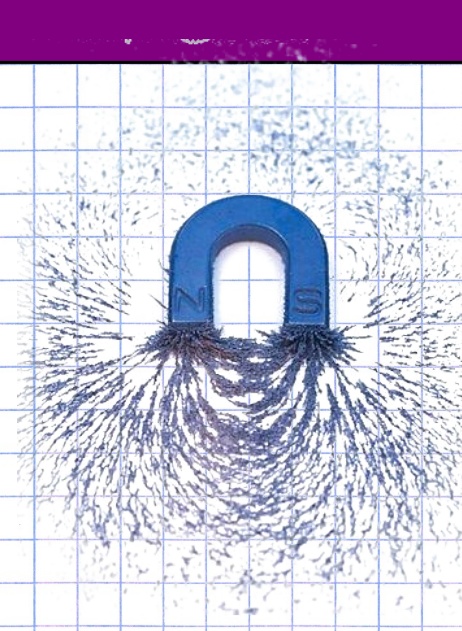
Towards
nucleation-propagation
and multidomain

FIG. 1. Particle size dependence of essentially spherical, randomly oriented, iron particles. Calculated curve given by solid line. Diameters $D = \hat{d}_v$. Data at 76°K obtained from electron microscopic examination ■, calculated from I_r/I_s vs temperature ○, and from smoothed data of H_{ci} vs D ●.

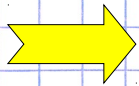
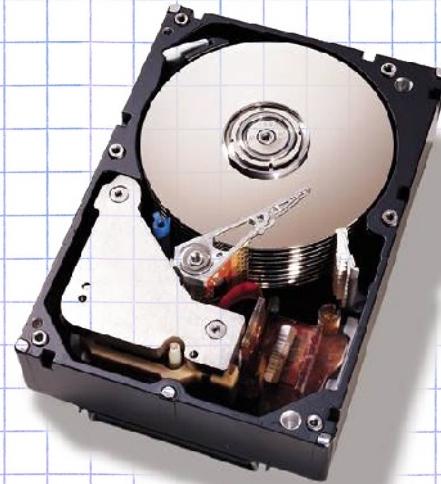
E. F. Kneller & F. E. Luborsky,
Particle size dependence of coercivity and remanence of single-domain particles,
J. Appl. Phys. 34, 656 (1963)



1. Fields, moments, materials



2. Magnetization processes



3. Thin film effects

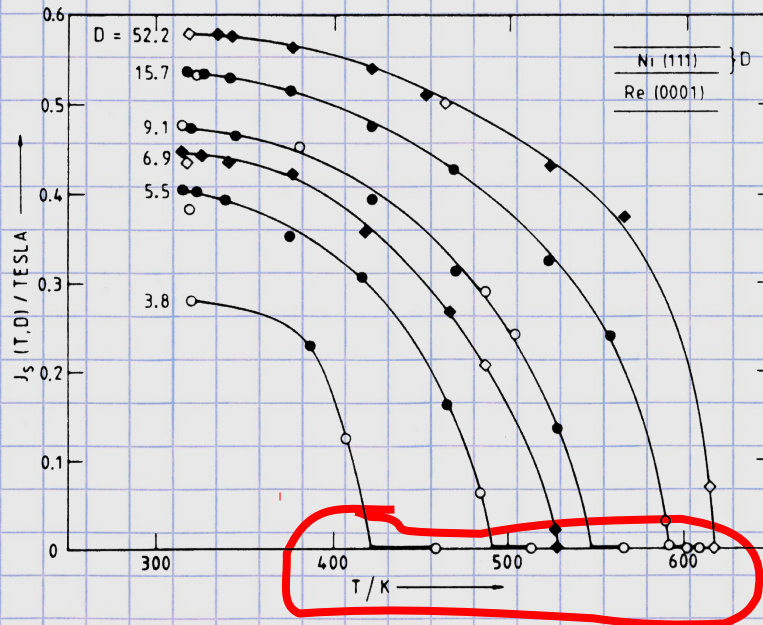
Elements of theory

- Ising (1925). No magnetic order at $T > 0K$ in 1D Ising chain.
- Bloch (1930). No magnetic order at $T > 0K$ in 2D Heisenberg. (spin-waves; isotropic Heisenberg)
- → N. D. Mermin, H. Wagner, PRL17, 1133 (1966)
- Onsager (1944) + Yang (1951).
2D Ising model: $T_c > 0K$

Magnetic anisotropy stabilizes ordering

Experiments

Ni(111)/Re(0001)



R. Bergholz and
U. Gradmann,
JMMM45, 389 (1984)

T_c interpreted with
molecular field

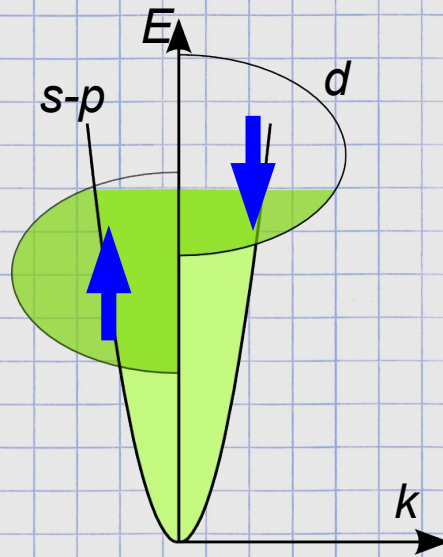
Noticeable
below $\approx 1nm$



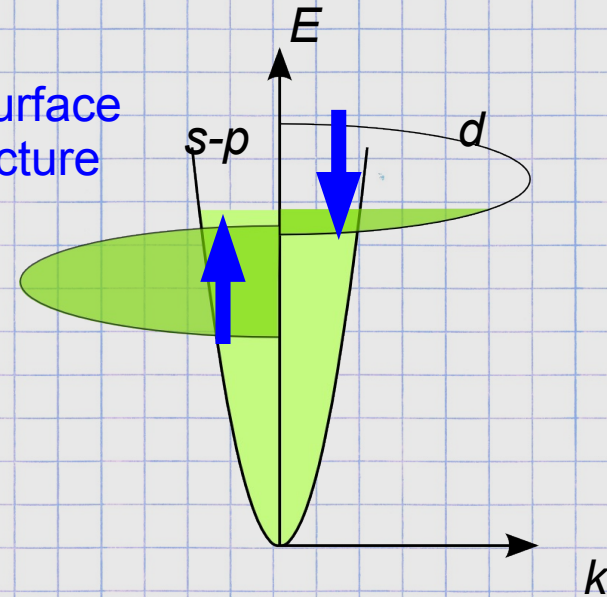
Simple picture: band narrowing at surfaces

Enhanced moment at surfaces

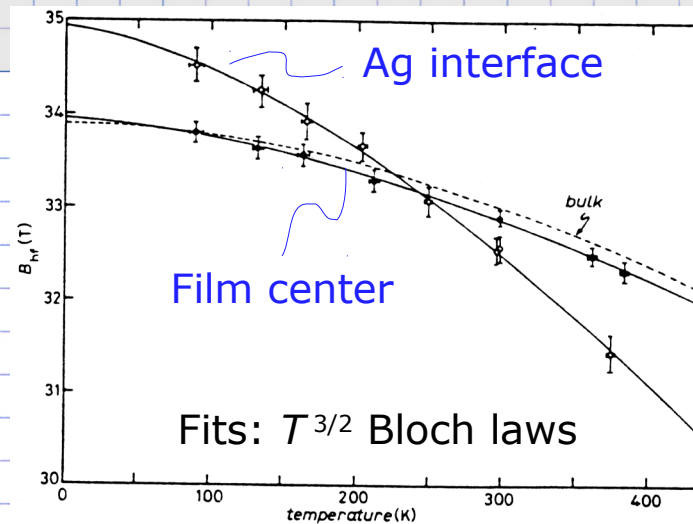
Bulk picture



Surface picture



In practice:
20-30 %, however
decays faster with
temperature



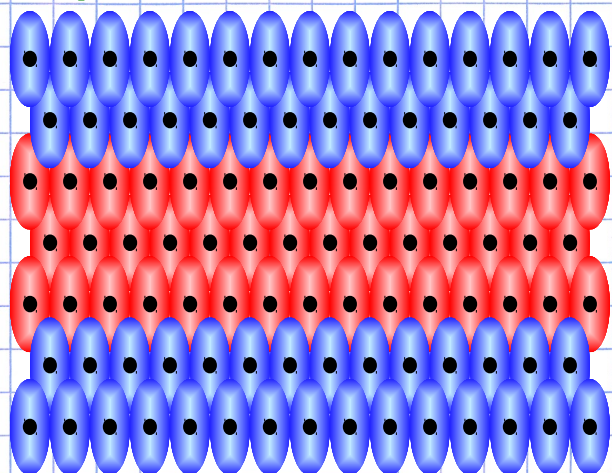
Ag/Fe(110)/W(110)

U. Gradmann et al.

Surface anisotropy

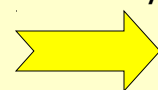
L. Néel,
J. Phys. Radium 15,
15 (1954)

« Superficial magnetic anisotropy and orientational superstructures »



Overview

Breaking of symmetry for
surface/interface atoms



Correction to the
magneto-crystalline energy

$$E_s = K_{s,1} \cos^2 \theta + K_{s,2} \cos^4 \theta + \dots$$

« This surface energy, of the order of 0.1 to 1 erg/cm², is liable to play a significant role in the properties of ferromagnetic materials spread in elements of dimensions smaller than 100Å »

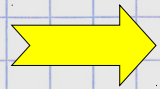
Pair model of Néel:

- Ks estimated from magneto-elastic constants
- Does not depend on interface material
- Yields order of magnitude only: correct value from experiments or calculations (precision !)

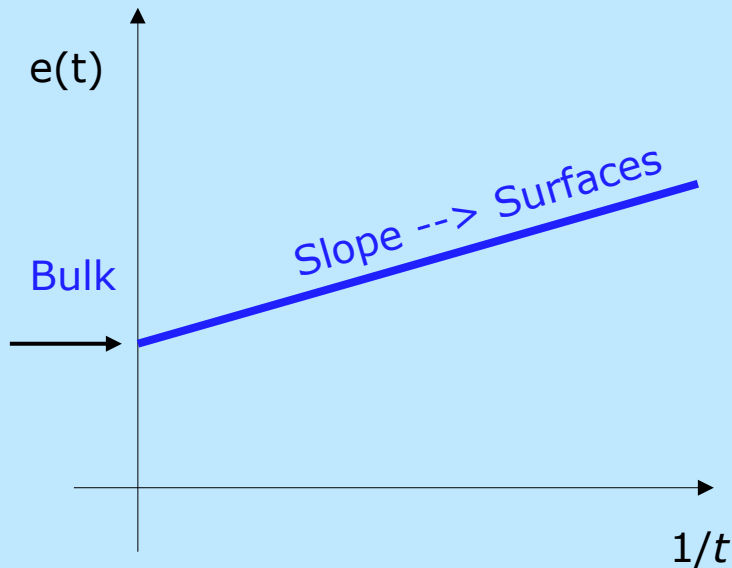


History of surface anisotropy : $1/t$ plot

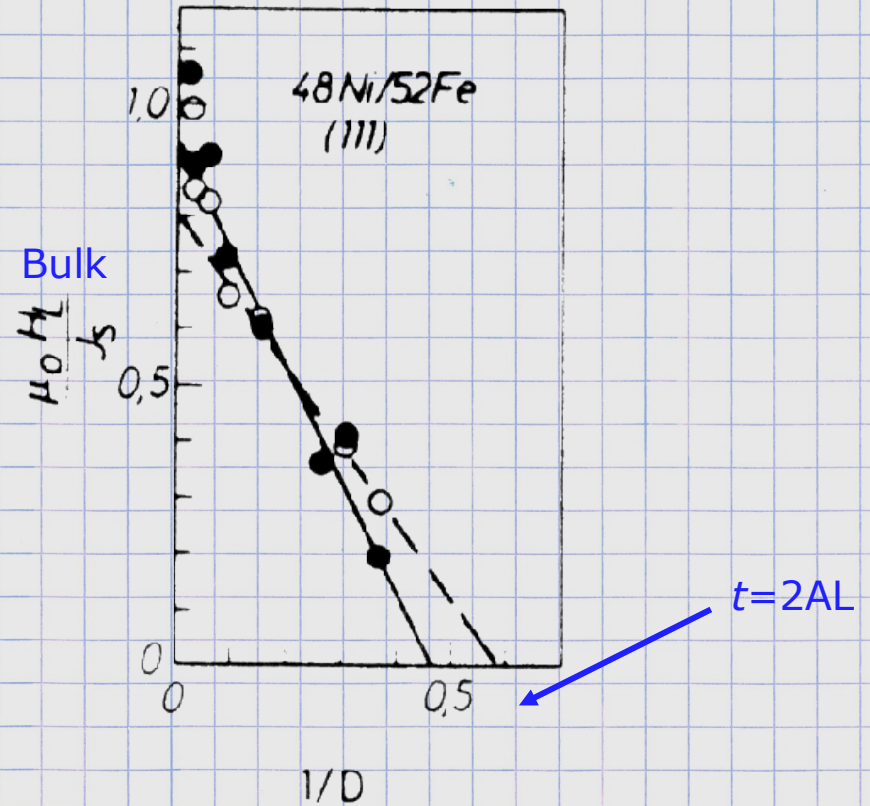
$$\varepsilon(t) = K_v t + 2K_s$$



$$E(t) = K_v + \frac{2K_s}{t}$$



First example of perpendicular anisotropy



U. Gradmann and J. Müller,
Phys. Status Solidi 27, 313 (1968)



What use ?

- ⇒ Well-defined anisotropy
 - do not need exact elliptic shape, better for down-scaling
- ⇒ High magnitude of anisotropy
 - better stability
- ⇒ Provides out-of-plane polarizer for spintronics
- ⇒ Etc.

Materials

- ⇒ 'Bulk-like magnetoelasticity : Co/Ni, NiPd etc.
- ⇒ Interface : 3d + heavy elements for large spin-orbit coupling : Co/Au, Co/Pt, Co/Pd
- ⇒ Interface (more recent)
 - 3d / oxides (MgO, Al₂O₃)
 - 3d / graphene

Figures

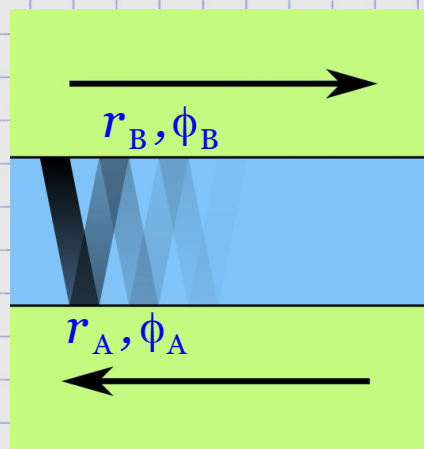
- ⇒ Interfaces 3d / heavy interfaces : up to 1-2nm
- ⇒ Interfaces 3d / oxide & graphene : up to 3-4nm
- ⇒ Multilayers : tens of nanometers

M. T. Johnson et al., Magnetic anisotropy in metallic multilayers, Rev. Prog. Phys. 59, 1409 (1996)

U. Gradmann, Magnetism in ultrathin transition metal films, in Handbook of Magnetism, K. H. J. Buschow (ed.), Elsevier Science Publishers, (1993)

The physics

Spin-dependent quantum confinement in the spacer layer



Forth & back phase shift

$$\Delta\phi = q t + \phi_A + \phi_B$$

Spin-independent

$$q = k^+ - k^-$$

Spin-dependent

$$r_A, \phi_A, r_B, \phi_B$$

⇒ Constructive or destructive interferences depending on spacer thickness

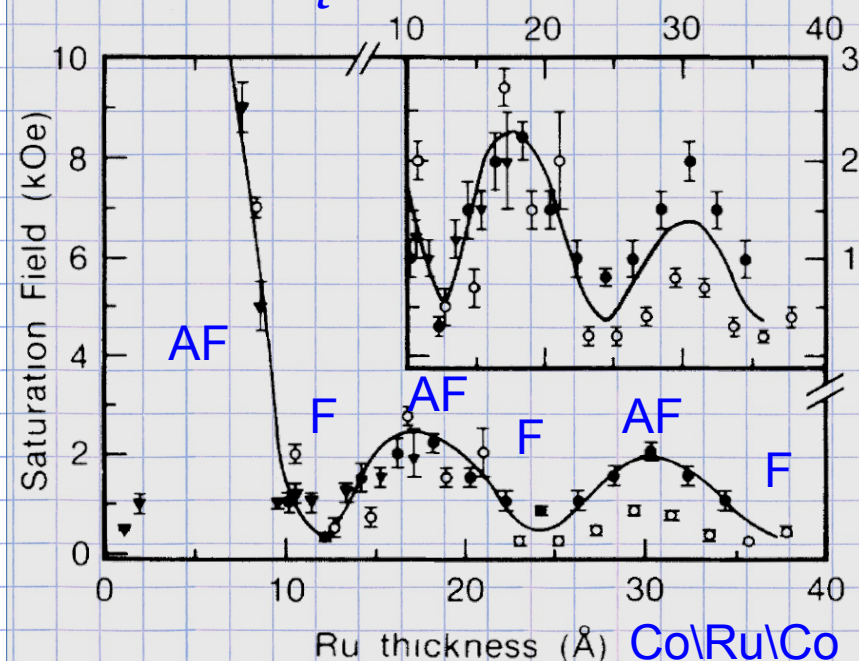
Illustration

Coupling strength:

$$E_s = J(t) \cos \theta \quad \text{in } \text{J/m}^2$$

$$\theta = \langle m_1, m_2 \rangle$$

$$\text{with: } J(t) = \frac{A}{t^2} \sin(q_\alpha t + \Psi)$$



S. S. P. Parkin et al., PRL64, 2304 (1990)

P. Bruno, J. Phys. Condens. Matter 11, 9403 (1999)



Ti	V	Cr	Mn	Fe	Co	Ni	Cu
No Coupling	9 0.1	3 9	7 .24	7 18			
2.89	2.62	2.50	2.24	2.48	2.50	2.49	2.56
Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag
No Coupling	9.5 .02	2.5 *	5.2 .12	3 5	3 11	7.9 1.6	3 9
3.17	2.86	2.72	2.71	2.65	2.69	2.75	2.89
Hf	Ta	W	Re	Os	Ir	Pt	Au
No Coupling	7 .01	2 *	5.5 .03	3 *	4.2 .41	3.5 10	
3.13	2.86	2.74	2.74	2.68	2.71	2.77	2.88

fcc bcc
 hcp complex cubic

Element
A_1 (Å)
ΔA_1 (Å)
J_1 (erg/cm ²)
P (Å)
r_{ws} (Å)

$$J(t) = \frac{A}{t^2} \sin\left(\frac{2\pi t}{P} + \psi\right)$$

Illustration of coupling strength

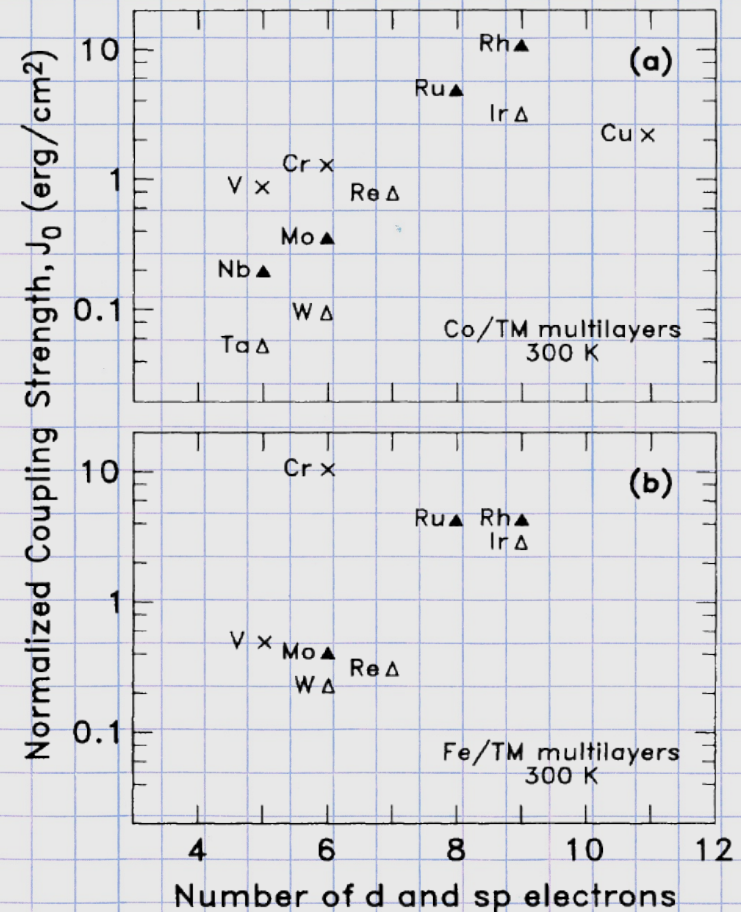


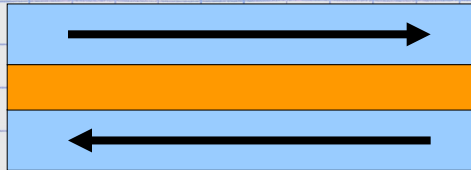
FIG. 3. Dependence of the normalized exchange coupling constant on the 3d, 4d and 5d transition metals in (a) Co/TM and (b) Fe/TM multilayers.

Note: $J(t)$ extrapolated for $t=3\text{\AA}$

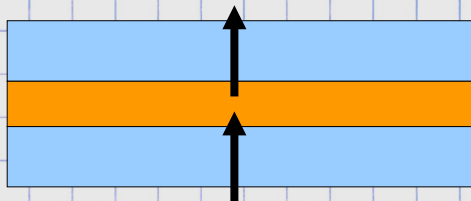
S. S. P. Parkin, Phys. Rev. Lett. 67, 3598 (1991)

Stacked dots : dipolar coupling

In-plane magnetization



Out-of-plane magnetization



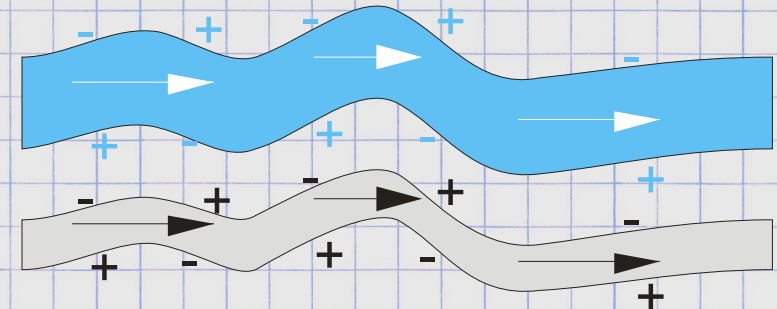
Hint:

An upper bound for the dipolar coupling is the self demagnetizing field

Notice: similar situation as for RKKY coupling

Stacked dots : orange-peel coupling

In-plane magnetization



Always parallel coupling

L. Néel, C. R. Acad. Sci. 255, 1676 (1962)
(valid only for thick films)

J. C. S. Kools et al., J. Appl. Phys. 85, 4466 (1999)
(valid for any films)

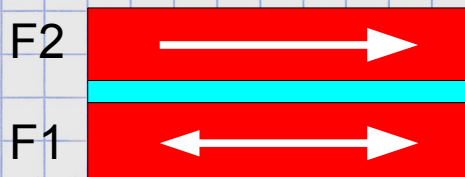
Out-of-plane magnetization

May be parallel or antiparallel

J. Moritz et al., Europhys. Lett. 65, 123 (2004)



Synthetic Ferrimagnets (SyF) — Crude description



Hypothesis:

- ⇒ Two layers rigidly coupled
- ⇒ Reversal modes unchanged
- ⇒ Neglect dipolar coupling

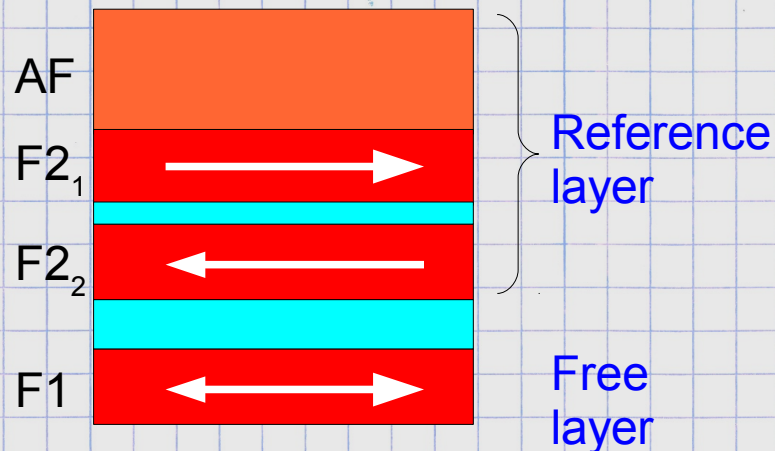


$$M = \frac{|e_1 M_1 - e_2 M_2|}{e_1 + e_2} \quad K = \frac{e_1 K_1 + e_2 K_2}{e_1 + e_2}$$

$$H_c \approx \frac{e_1 M_1 H_{c,1} + e_2 M_2 H_{c,2}}{|e_1 M_1 - e_2 M_2|}$$

What use?

- ⇒ Increase coercivity of layers
- ⇒ Decrease intra- and inter- dot dipolar coupling



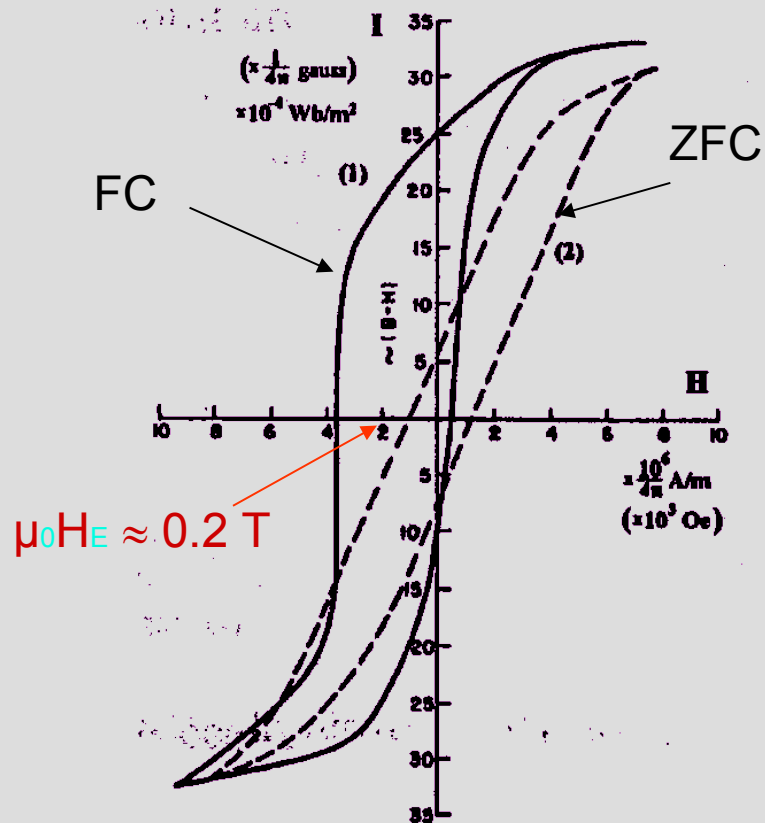
Practical aspects

- ⇒ Ru spacer layer (largest effect)
- ⇒ Control thickness within a few Angströms !

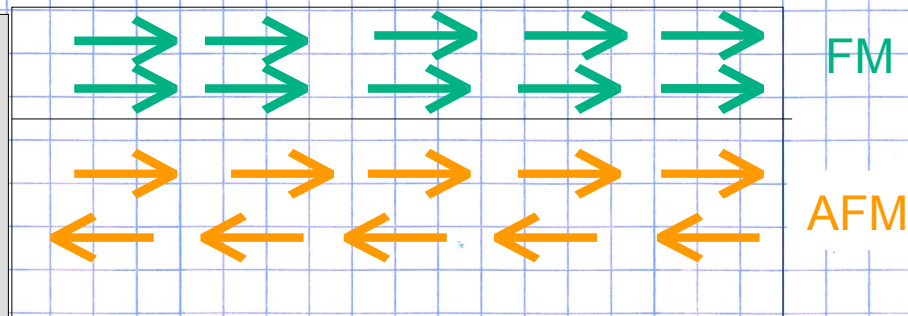


Seminal studies

Oxidized Co nanoparticles



Meiklejohn and Bean,
Phys. Rev. 102, 1413 (1956),
Phys. Rev. 105, 904, (1957)



Field-cooled hysteresis loops

↻ Shift in field
↻ Increase coercivity

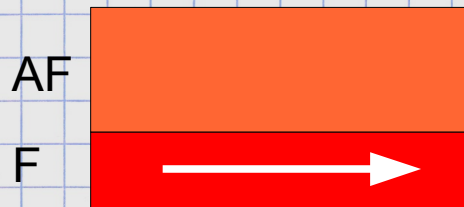
Exchange bias

J. Nogués and Ivan K. Schuller
J. Magn. Magn. Mater. 192 (1999) 203

Exchange anisotropy—a review

A E Berkowitz and K Takano
J. Magn. Magn. Mater. 200 (1999)

Increase coercivity of layers



Crude approximation for thin layers:

$$H_{F-AF} \approx H_F \left(1 + \frac{K_{AF} t_{AF}}{K_F t_F} \right)$$

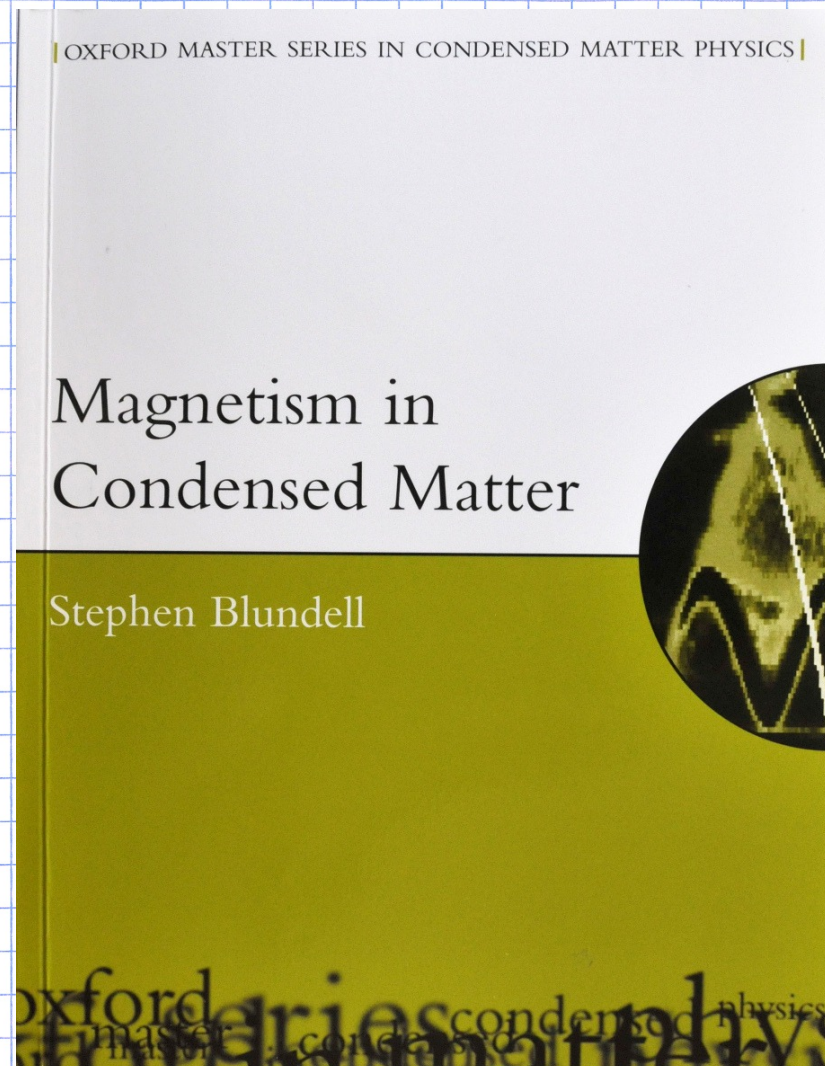
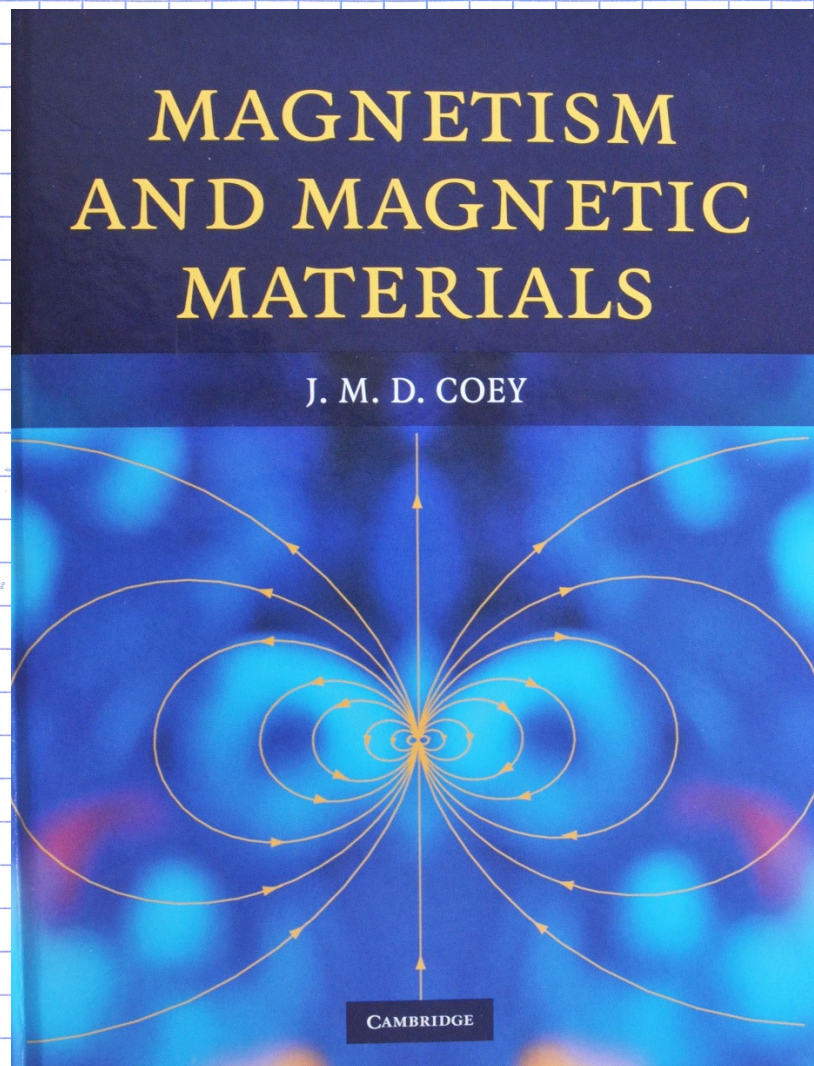
Application

Concept of spin-valve in magneto-resistive elements



B. Diény et al., Phys. Rev. B 43, 1297 (1991)

- ⇒ Sensors
- ⇒ Memory cells
- ⇒ Etc.



Repository of lectures of the European School on Magnetism: <http://magnetism.eu>



- [1] Magnetic domains, A. Hubert, R. Schäfer, Springer (1999, reed. 2001)
- [2] R. Skomski, Simple models of Magnetism, Oxford (2008).
- [3] R. Skomski, Nanomagnetism, J. Phys.: Cond. Mat. 15, R841–896 (2003).
- [4] O. Fruchart, A. Thiaville, Magnetism in reduced dimensions, C. R. Physique 6, 921 (2005) [Topical issue, Spintronics].
- [5] Lecture notes from undergraduate lectures, plus various slides:
<http://perso.neel.cnrs.fr/olivier.fruchart/slides/>
- [8] J.I. Martin et coll., Ordered magnetic nanostructures: fabrication and properties, J. Magn. Magn. Mater. 256, 449-501 (2003)
- [9] Lecture notes in magnetism: <http://magnetism.eu/esm/repository.html>



Moment and anisotropy of ultrathin films

U. Gradmann, Handbook of magnetic materials vol. 7, K. H.K. Buschow Ed., Elsevier, Magnetism of transition metal films, 1 (1993)

M. Farle, Ferromagnetic resonance of ultrathin metallic layers, Rep. Prog. Phys. 61, 755 (1998)

P. Pouloupoulos et al., K. Baberschke, Magnetism in thin films, J. Phys.: Condens. Matter 11, 9495 (1999)

H. J. Elmers, Ferromagnetic Monolayers, Int. J. Mod. Phys. B 9 (24), 3115 (1995)

O. Fruchart, Epitaxial self-organization: from surfaces to magnetic materials, C. R. Phys. 6, 61 (2005)

O. Fruchart et al., Magnetism in reduced dimensions, C. R. Phys. 6, 921 (2005)

Perpendicular anisotropy

M. T. Johnson et al., Magnetic anisotropy in metallic multilayers, Rep. Prog. Phys. 59, 1409 (1996)

Exchange-bias

J. Nogues et al., I. K. Schuller, Exchange bias, J. Magn. Magn. Mater 192 (2), 203 (1999).

Magneto-elasticity in thin films

D. Sander, The correlation between mechanical stress and magnetic anisotropy in ultrathin films, Rep. Prog. Phys. 62, 809 (1999)

Theory (misc)

T. Asada et al., G. Bihlmayer, S. Handschuh, S. Heinze, P. Kurz, S. Blügel, First-principles theory of ultrathin magnetic films, J. Phys.: Condens. Matter 11, 9347 (1999)

F. J. Himpsel et al., Magnetic Nanostructures, Adv. Phys. 47 (4), 511 (1998)

P. Bruno, Theory of interlayer exchange interactions in magnetic multilayers, J. Phys.: Condens. Matter 11, 9403 (1999)