Introduction to low-dimensional magnetism

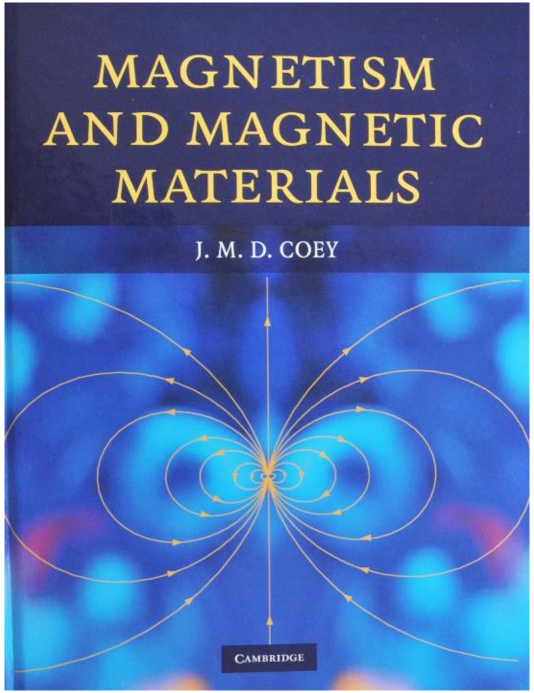
Olivier FRUCHART

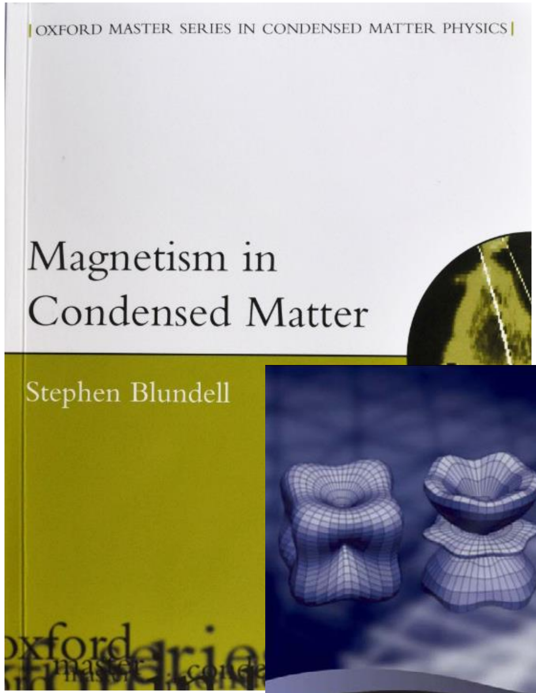
University Grenoble Alpes / CNRS / CEA / G-INP, SPINTEC, France

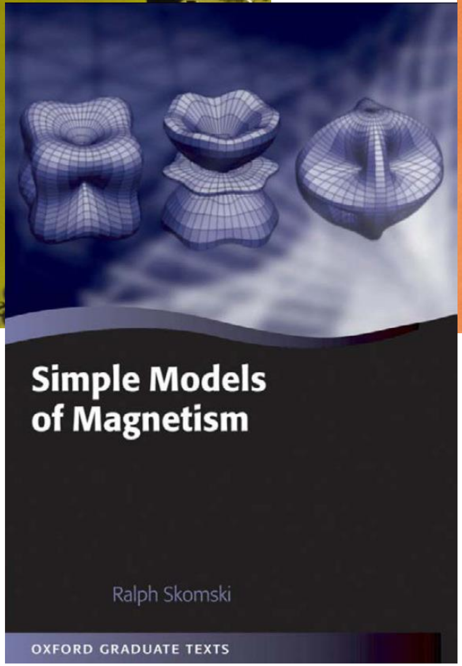
Olivier.fruchart@cea.fr

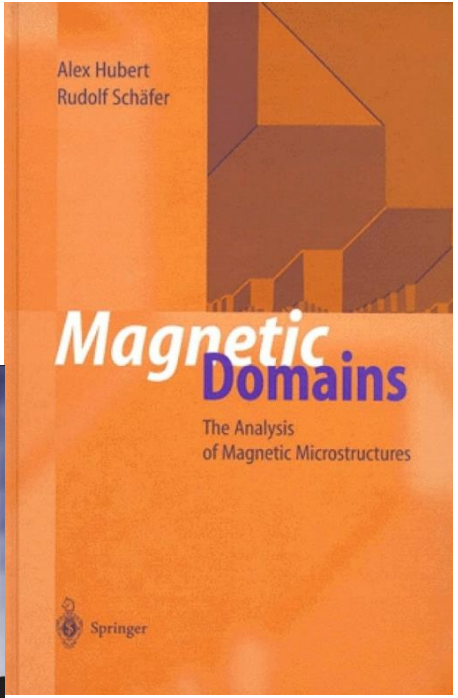
Slides: http://fruchart.eu/slides

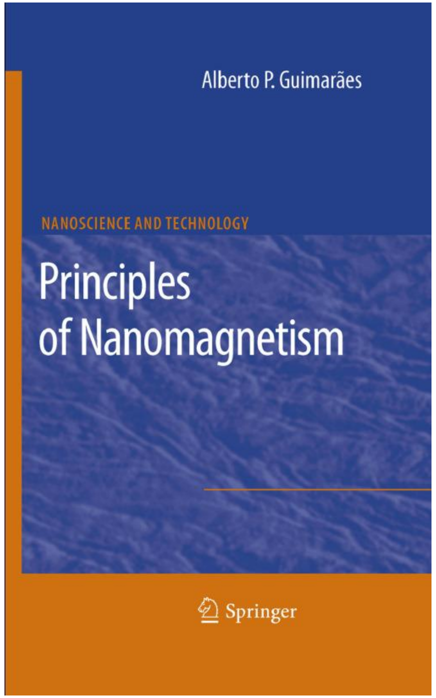
Some references – Books











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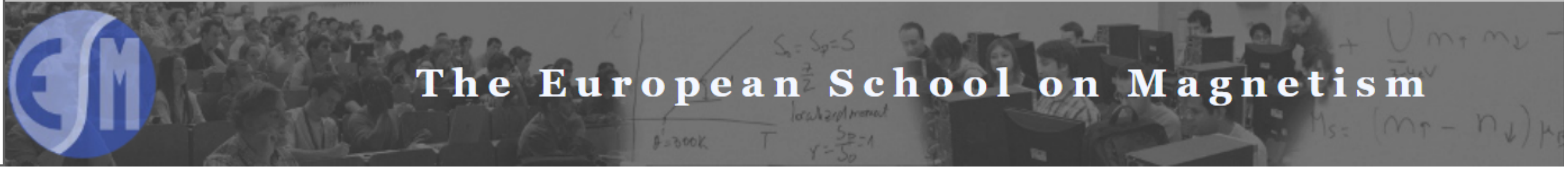

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The lectures of all ESM schools since 2003 are ordered here in terms of topics. Those pertaining to several topics are listed several times. The topics are:

Magnetic field and moments

- **[2020]** Origin of magnetism (spin and orbital momentum, atoms and ions, paramagnetism and diamagnetism): **STEPHEN BLUNDELL**, *Oxford, UK* [Slides | Recording]
- **[2020]** Fields, moments, units: **OLIVIER FRUCHART**, *Grenoble, France* [Slides | Recording]
- **[2019]** Fields, moments, units: **OLIVIER FRUCHART**, *Grenoble, France* [Abstract | Slides]
- **[2019]** Magnetism of atoms, Hund's rules, spin-orbit in atoms: **VIRGINIE SIMONET**, *Grenoble, France* [Abstract]
- **[2018]** Units in Magnetism (practical): **OLIVIER FRUCHART**, *Grenoble, France* [Questions | Answers]
- **[2018]** Magnetism of atoms and ions: **JANUSZ ADAMOWSKI**, *Kraków, Poland* [Abstract | Slides]
- **[2018]** Fields, Moments, Units, Magnetostatics: **RICHARD EVANS**, *York, UK* [Abstract | Slides]
- **[2017]** Fields, Units, Magnetostatics: **LAURENT RANNO**, *Grenoble, France* [Abstract | Slides]
- **[2017]** Magnetism of atoms and ions: **WULF WULFHEKEL**, *Karlsruhe, Germany* [Abstract | Slides]
- **[2017]** Units in Magnetism (practical): **OLIVIER FRUCHART**, *Grenoble, France* [Questions | Answers]
- **[2015]** Units in Magnetism (practical): **OLIVIER FRUCHART**, *Grenoble, France* [Questions | Answers]

Topics

- Units, fields and moments
- Exchange, magnetic ordering, magnetic anisotropy
- Temperature effects and excitations
- Correlated systems
- Transport
- Magnetization processes
- Simulations
- Materials
- Nanoparticles, microstructures etc
- Nanomagnetism and spintronics
- Techniques
- Applications and interdisciplinary magnetism
- Industry perspectives
- Open sessions

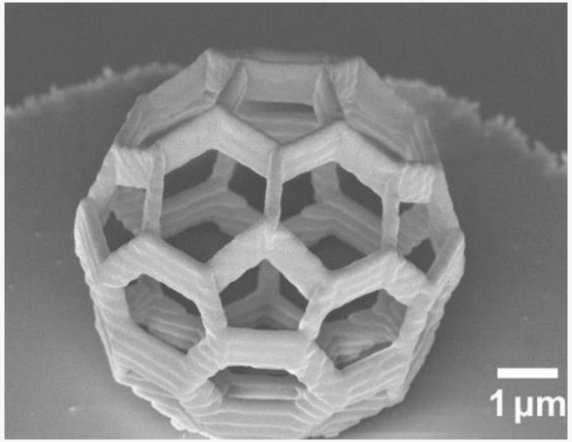
Note: OF lecture in 2003 on low-dimensional magnetism

General considerations – What is dimensionality?

Space

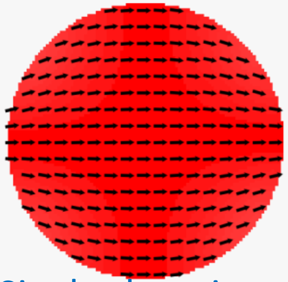
- Physical quantities (incl. magnetization) defined at any location in space

$$\mathbf{M} = \mathbf{M}(x, y, z)$$

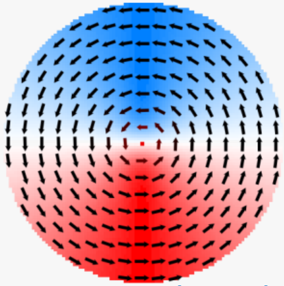


C. Donnnelly, Phys. Rev. Lett. 114, 115501 (2015)

- Features: shape, dimensions, dimensionality



Single-domain

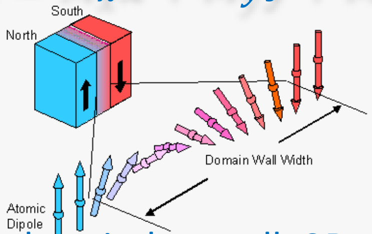


Size > magn. length scale

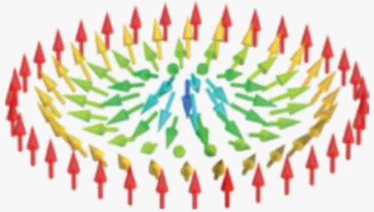
Magnetization components

- Vector field for magnetization has three components

$$\mathbf{M} = M_x \hat{\mathbf{x}} + M_y \hat{\mathbf{y}} + M_z \hat{\mathbf{z}}$$



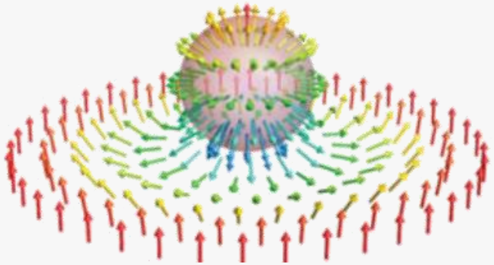
Simple spiral or wall: 2D



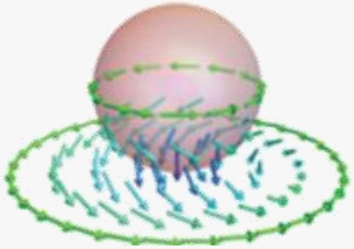
Skyrmion: 3D

A. Fert, Nat. Nanotech. 8, 152 (2013)

- Mapping magnetization on the unit sphere



Skyrmion

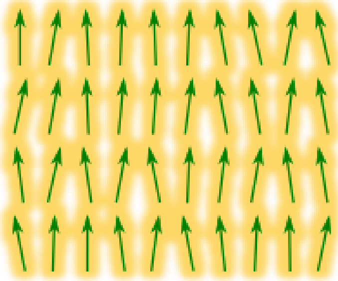


Vortex

In: H.B. Braun, *Solitons in real space: domain walls, vortices, hedgehogs and skyrmions*, Springer (2018)

Table of contents

- Magnetic ordering



Magnetic exchange

Physics

- Spin + space wave function must be antisymmetric
- Coulomb repulsion
- Pauli exclusion

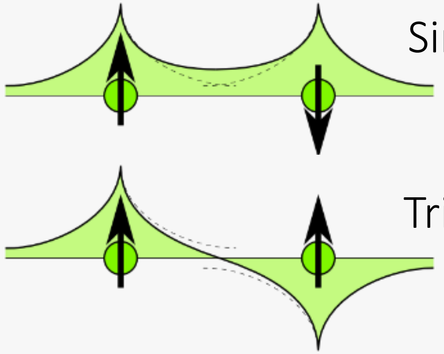
→ May be viewed as interatomic Hund's rules

Hamiltonian

$$\mathcal{H} = -2J_{1,2}\mathbf{S}_1 \cdot \mathbf{S}_2$$

$J_{1,2}$ Exchange integral

Direct exchange



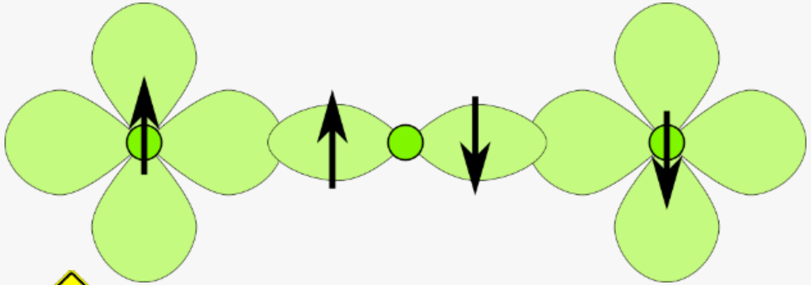
Singlet

Triplet

Molecules → Singlet

Metals → Ferro/Antiferro

Superexchange



Bond-length and – orientation dependent

Often: π → Antiferro; $\pi/2$ → ferro

Indirect exchange

Conduction electrons

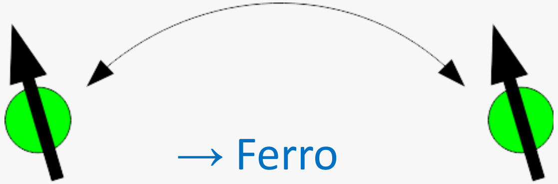
→ Ferro

RKKY, rare-earth (4f), GaMnAs (3d)

Double exchange

Mixed-valence states

e- hopping



→ Ferro

Mn^{3+} ($3d^4$) Mn^{4+} ($3d^3$)

Example: $(La_{0.7}Ca_{0.3})MnO_3$

Magnetic ordering and orders

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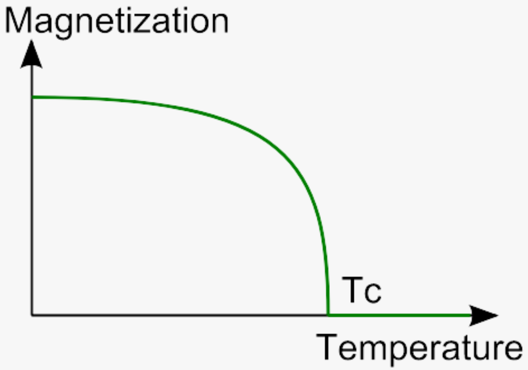
Magnetic ordering

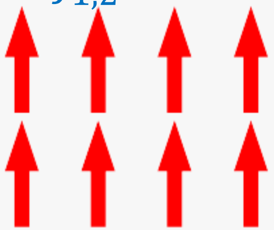
Magnetic exchange between microscopic moments:

$$\mathcal{E} = -2 \sum_{i<j} J_{1,2} \mathbf{S}_i \cdot \mathbf{S}_j$$



Ferromagnetism

$J_{1,2} > 0$



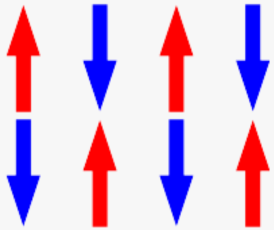
Fe

$T_C = 1043$ K

$M_s = 1.73 \times 10^6$ A/m

Antiferromagnetism

$J_{1,2} < 0$



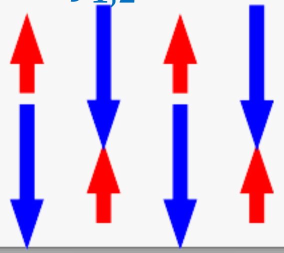
CoO

$T_N = 292$ K

$J = 3/2$

Ferrimagnetism

$J_{1,2} < 0$



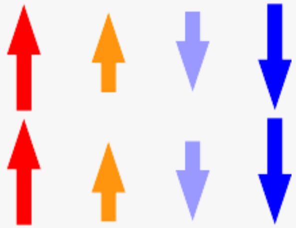
Fe_3O_4

$T_C = 858$ K

$M_s = 480$ kA/m

Helimagnetism

$J_1, J_2 \dots$



Dy

$T \in 85 - 179$ K

$\mu = 10.4\, \mu_B$

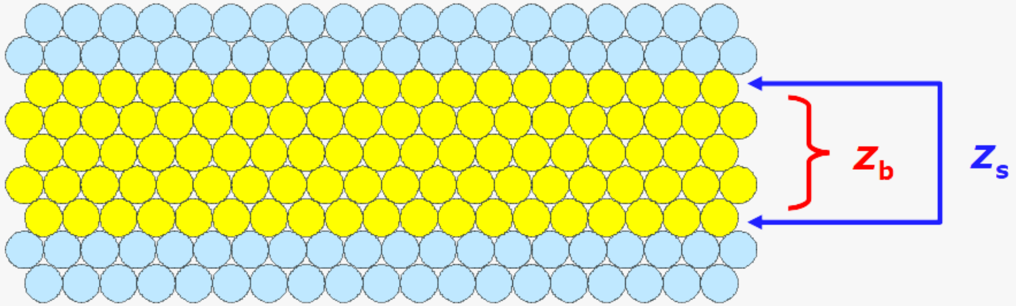
Ordering and dimensionality (theory)

A bit of theory

- **Ising (1925)**. No magnetic order at T>0K in 1D Ising chain.
- **Bloch (1930)**. No magnetic order at T>0K in 2D Heisenberg (spin-waves)
  N. D. Mermin, H. Wagner, PRL17, 1133 (1966)
- **Onsager (1944) + Yang (1951)**. 2D Ising model: Tc>0K

➡ Magnetic anisotropy promotes ordering

Naïve views: mean field



$$T_C = \frac{\mu_0 z n_{W,1} n g_J^2 \mu_B^2 J(J+1)}{3k_B}$$

N atomic layers ➡ $\langle z \rangle = z_b - \frac{2(z_b - z_s)}{N}$

nearest neighbors

➡ $\Delta T_C(t) \sim 1/t$

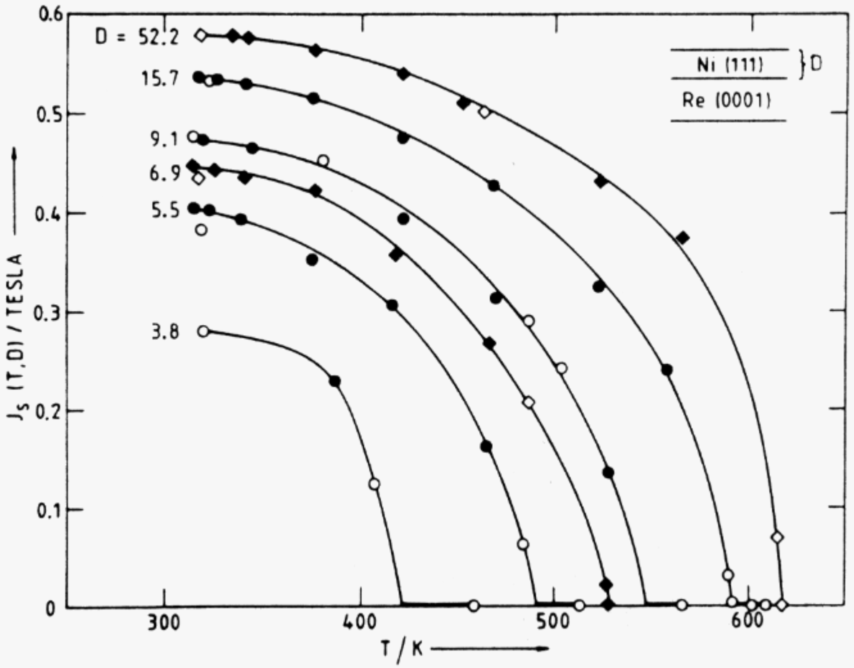
Confirmed by a more robust layer-dependent mean-field theory

G.A.T. Allan, PRB1, 352 (1970)

Ordering and dimensionality (experiments)

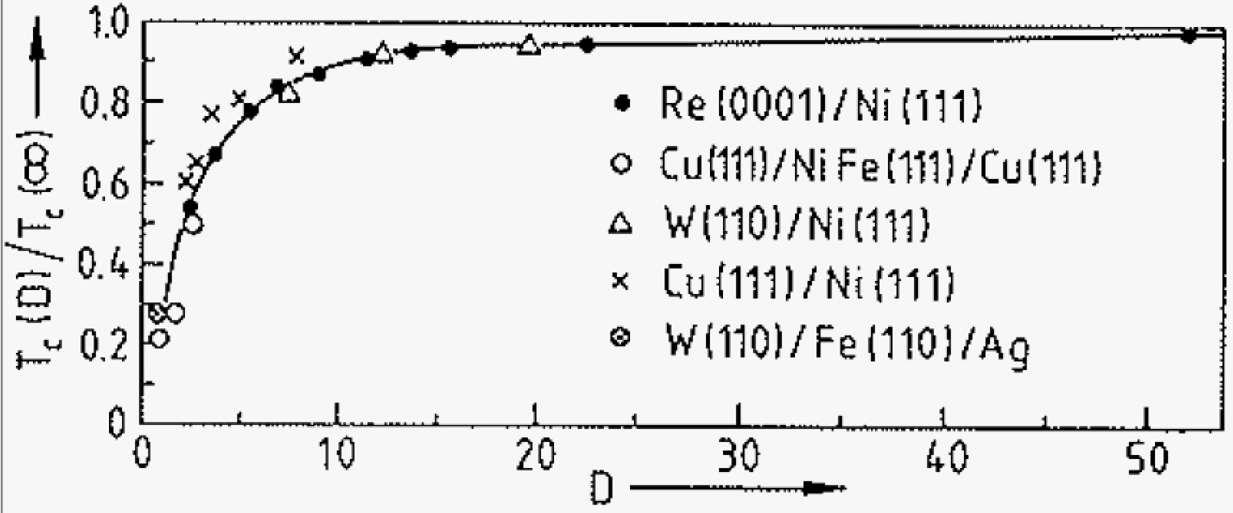
Qualitative

Ni(111)/Re(0001)



R. Bergholz and U. Gradmann, J. Magn. Magn. Mater. 45, 389 (1984)

Quantitative, master curve

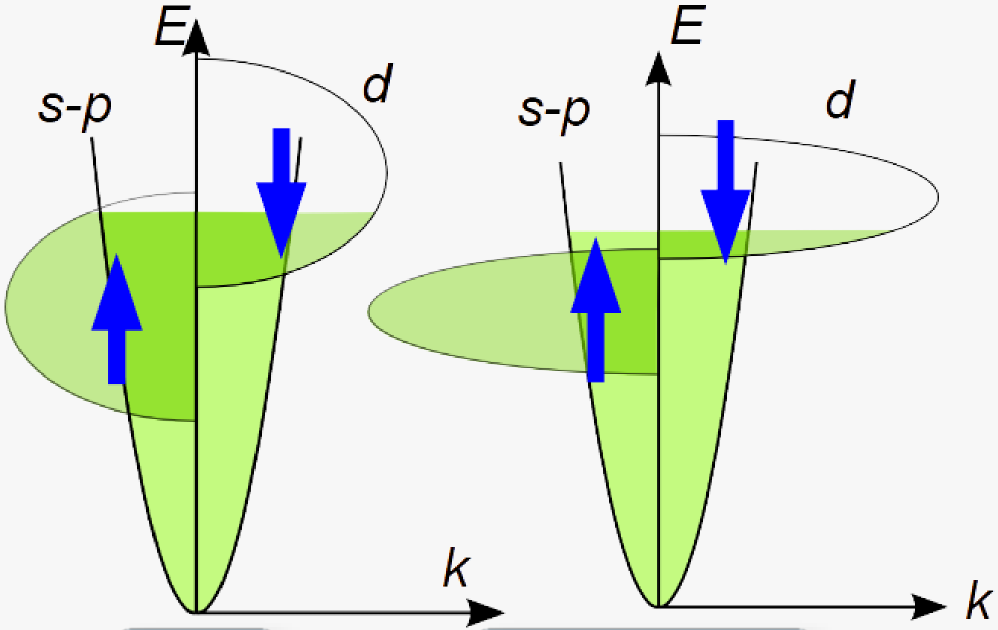


- Curie temperature well fitted by molecular field $\Delta T_C(t) \sim 1/t$

- Ordering temperature decreases with thickness
- Very significant below ≈ 1nm

Magnetic moment versus dimensionality

Simple picture: band narrowing at surfaces

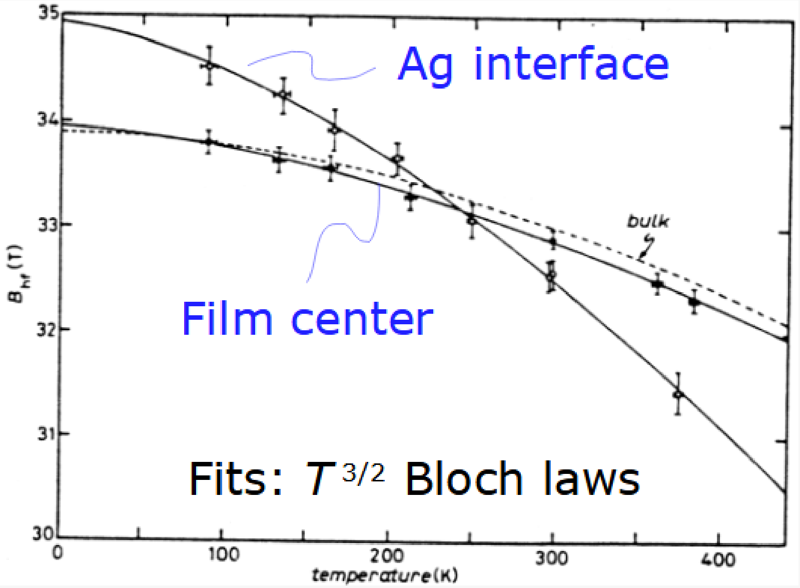


Bulk

At a surface

In practice

Ag/Fe(110)/W(110)

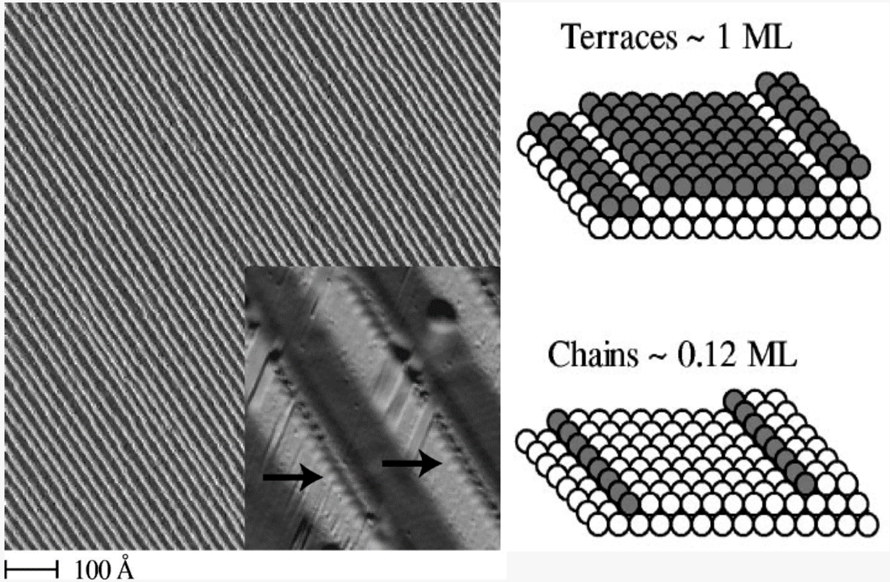


U. Gradmann et. al.

- Surface moments are usually 20-30% larger than in the bulk
- However, decay faster with temperature

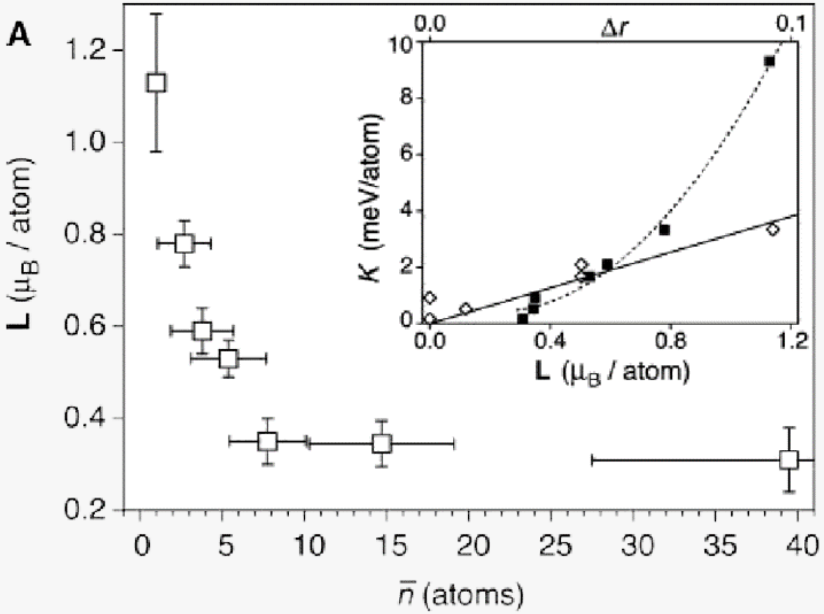
Magnetic moment versus dimensionality

Example of system: sub-monolayer Co/Pt(997)



A. Dallmeyer et al., Phys.Rev.B 61(8), R5153 (2000)

Example of system: Co/Pt(111)



P. Gambardella et al., Science 300, 1130 (2003)

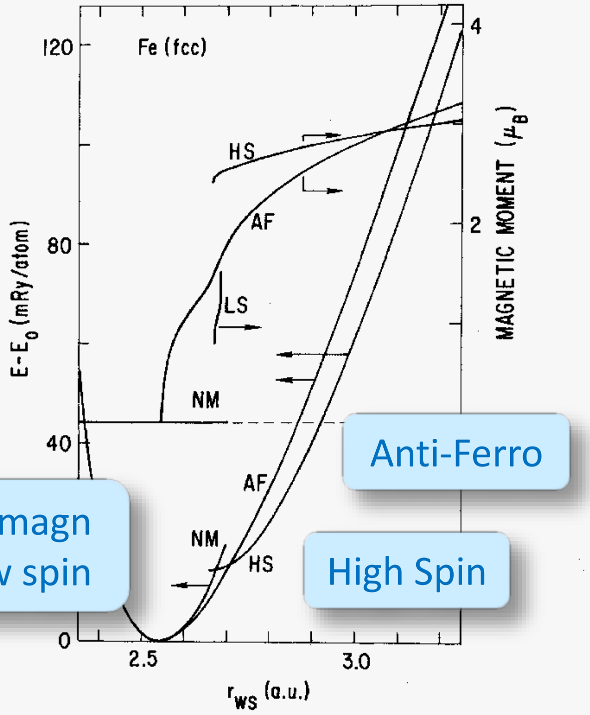
Conclusions

- From bulk to atoms: considerable increase of orbital moment
- 2 atoms closer to wire than 1 atom
- bi-atomic wire closer to surface than wire

Magnetic order versus dimensionality – Example: Fe

Theory (bulk)

- fcc γ-Fe for T>1185K: non-magnetic
- 'ground-state': sensitive on strain

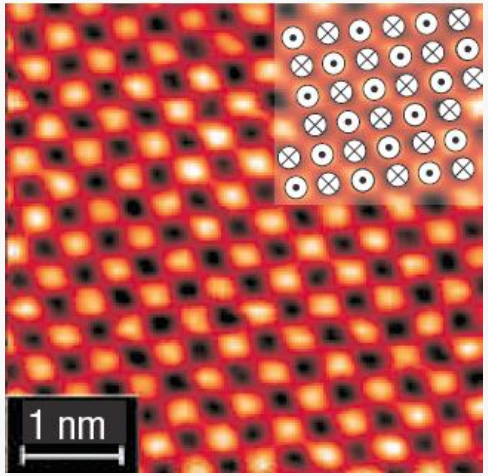


V. L. Moruzzi et al., PRB39, 6957 (1989)

See also: O.K. Andersen, Physica B 86, 249 (1977)

(Some) experiments in low dimensions

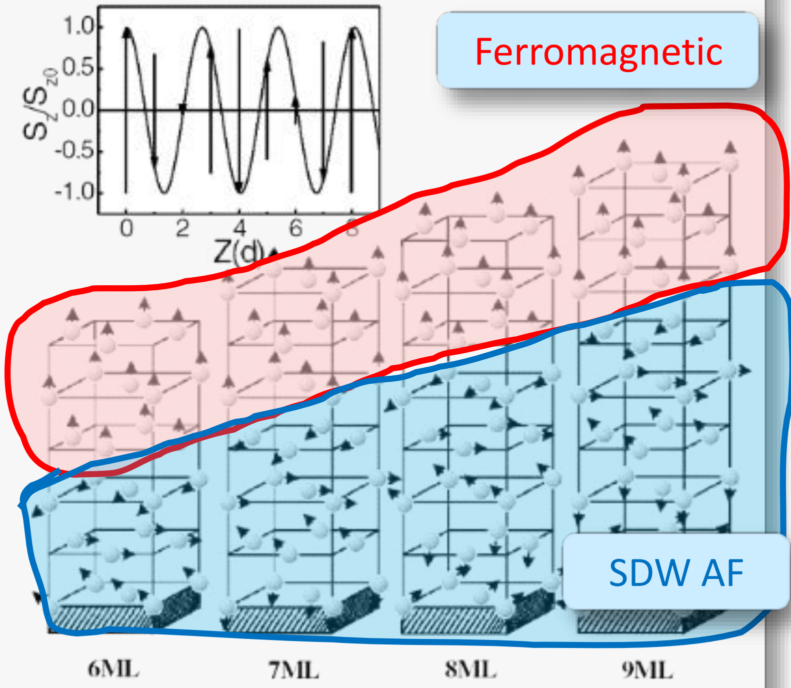
AF Fe(1ML)/W(001)



Antiferromagnetic domain (SP-STM)

M. Bode et al., Nat. Mater. 5, 477-481 (2006)

Spin-density-wave AF Fe(001)

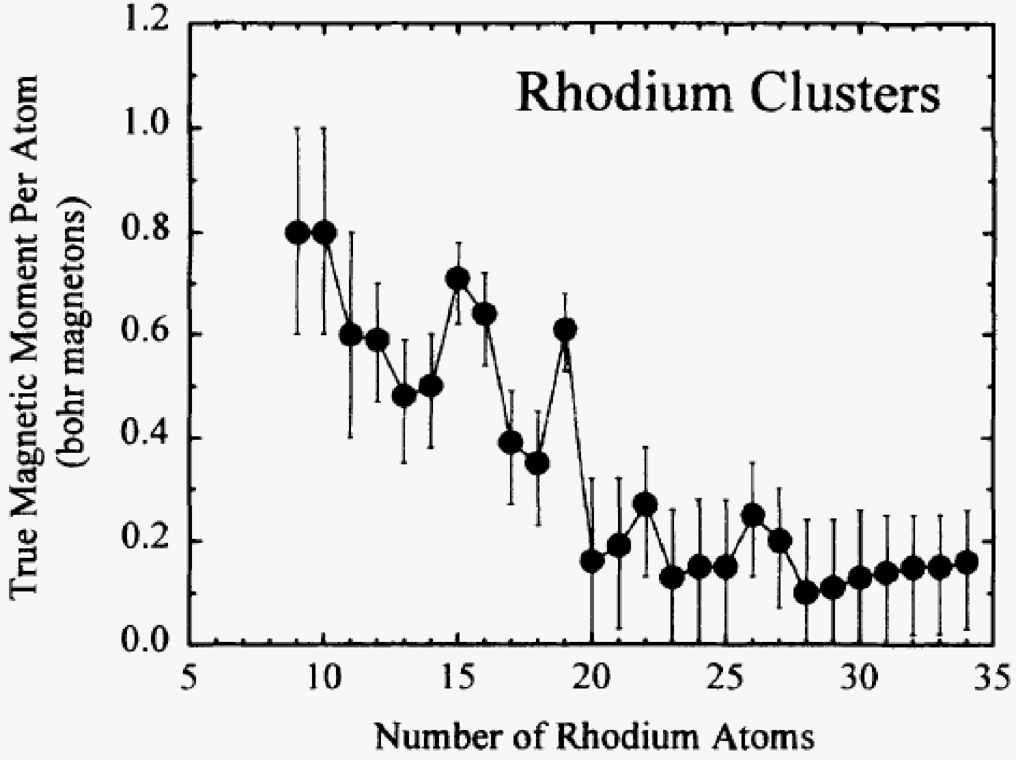


D. Qian et al., PRL87, 227204(2001)

See also V. Cros et al., Europhys. Lett. 49, 807 (2000)

Magnetic order versus dimensionality – Example: Fe

Rh turning magnetic in small clusters



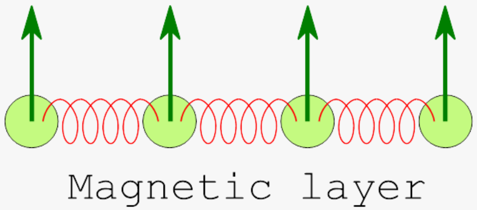
A. J. Cox et al., Magnetism in 4d-transition metal clusters, Phys. Rev. B 49, 12295 (1994)

Physics
Narrowing of band
Stoner criterium
Quantum size effects

Chirality

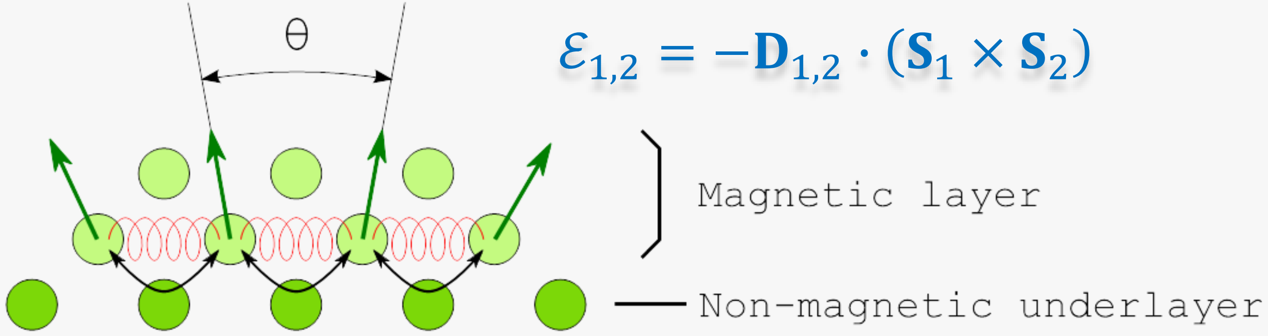
Magnetic exchange

$$\mathcal{E}_{1,2} = -J_{1,2}\,\mathbf{S}_1 \cdot \mathbf{S}_2$$

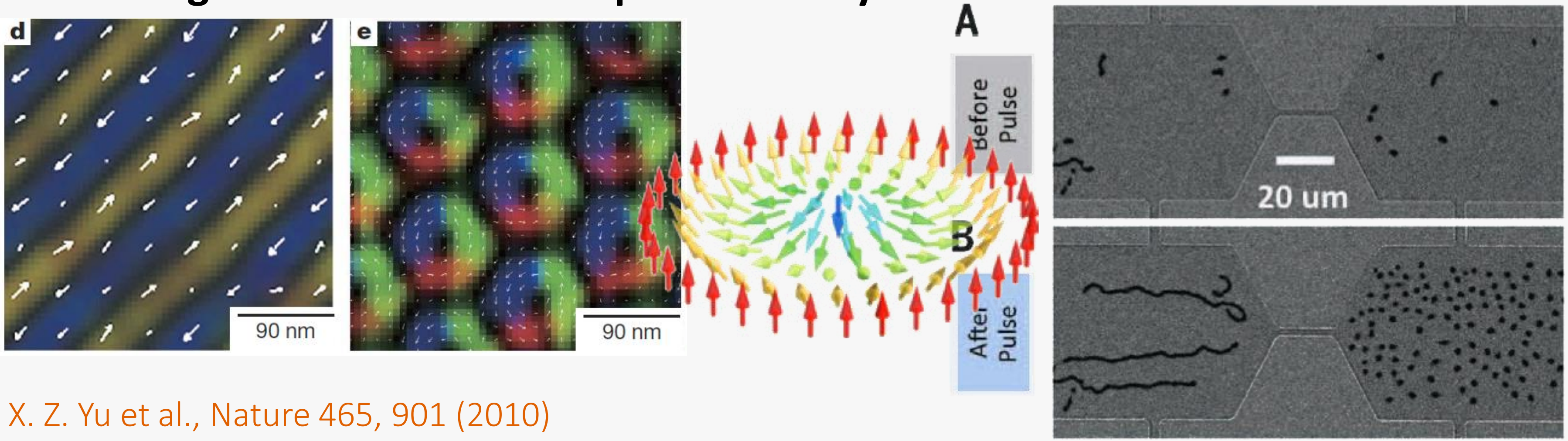


The Dzyaloshiinski-Moriha interaction (DMI)

$$\mathcal{E}_{1,2} = -\mathbf{D}_{1,2} \cdot (\mathbf{S}_1 \times \mathbf{S}_2)$$



Chiral magnetization textures: spirals and skyrmions

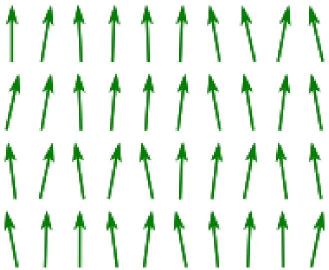


W. Jiang et al., Science 349, 283 (2015)

X. Z. Yu et al., Nature 465, 901 (2010)

Table of contents

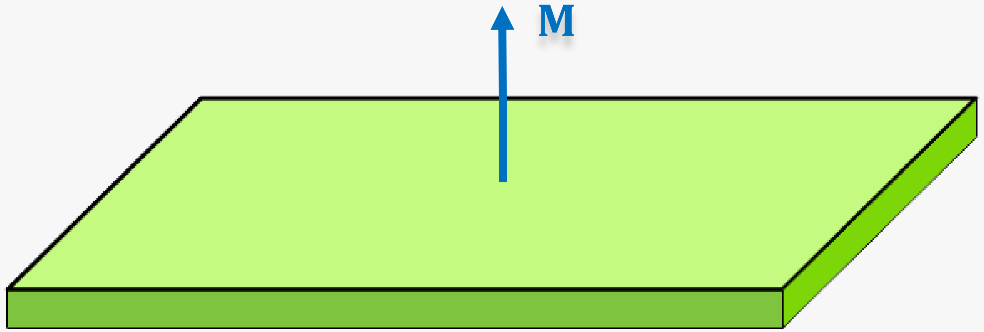
- Magnetic ordering



- Magnetic anisotropy

Quizz 1 – Stray field above thin film

What is the value of stray field above an extended perpendicularly-magnetized thin film?

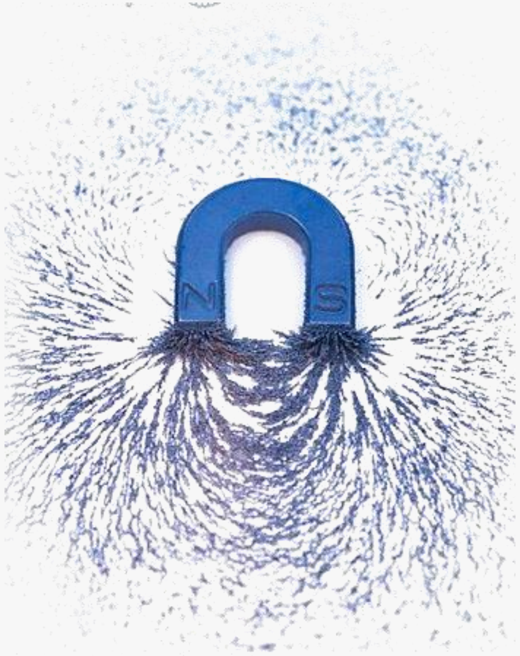


1. Zero
2. + M / 2
3. + M
4. Depends on the value of anisotropy

Quizz 2 – Maximum stray field

What is the maximum stray field (in free space) that can be obtained from a uniformly-magnetized system of arbitrary shape?

1. $M/3$
2. $M/2$
3. M
4. As high as we like, no limit

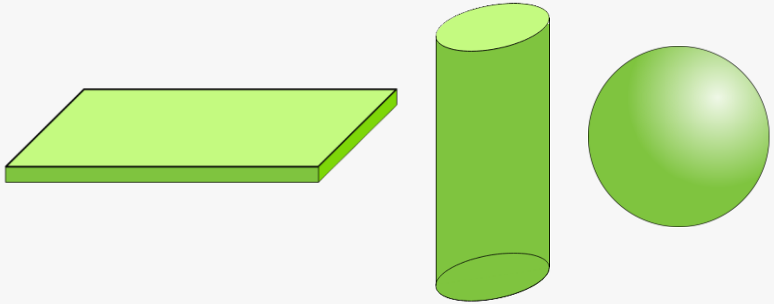


Quizz 3 – Demagnetizing coefficients

QUIZ!

One can define and compute (analytics or numerics) demagnetizing coefficients for…

1. Ellipsoids, slabs, cylinders



2. Any finite-size body made up of one single piece

3. Any finite-size body of arbitrary shape

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4. None, all is an approximation

Magnetostatics – Ways to handle it

Analogy with electrostatics

Maxwell equation → $\boldsymbol{\nabla} \cdot \mathbf{H}_\mathrm{d} = -\boldsymbol{\nabla} \cdot \mathbf{M}$

$$\mathbf{H}_\mathrm{d}(\mathbf{r}) = -M_\mathrm{s} \iiint_{\mathcal{V}'} \frac{[\boldsymbol{\nabla} \cdot \mathbf{m}(\mathbf{r}')]\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3} \mathrm{d}\mathcal{V}'$$

To lift the singularity that may arise at boundaries, a volume integration around the boundaries yields:

$$\mathbf{H}_\mathrm{d}(\mathbf{r}) = \iiint \frac{\rho(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3} \mathrm{d}\mathcal{V}' + \oiint \frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3} \mathrm{d}\mathcal{S}'$$

Magnetic charges

$\rho(\mathbf{r}) = -M_\mathrm{s}\, \boldsymbol{\nabla} \cdot \mathbf{m}(\mathbf{r})$ → volume density of magnetic charges

$\sigma(\mathbf{r}) = M_\mathrm{s}\, \mathbf{m}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r})$ → surface density of magnetic charges

Useful expressions

$$\mathcal{E}_\mathrm{d} = -\frac{1}{2}\mu_0 \iiint_{\mathcal{V}} \mathbf{M} \cdot \mathbf{H}_\mathrm{d}\, \mathrm{d}\mathcal{V}$$

$$\mathcal{E}_\mathrm{d} = \frac{1}{2}\mu_0 \iiint_{\mathcal{V}} \mathbf{H}_\mathrm{d}^2\, \mathrm{d}\mathcal{V}$$

$$\mathcal{E}_\mathrm{d} = \frac{1}{2}\mu_0 \left(\iiint_{\mathcal{V}} \rho\phi\, \mathrm{d}\mathcal{V} + \oiint \sigma\phi\, \mathrm{d}\mathcal{S} \right)$$

- Always positive
- Zero means minimum

Magnetostatics – Size and shape

Size considerations

$$\mathbf{H}_\mathrm{d}(\mathbf{r}) = \iiint \frac{\rho(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{V}' + \oint\!\!\oint \frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$$

- Unchanged if all lengths are scaled: homothetic.
  NB: the following is a solid angle:

$$\mathrm{d}\mathbf{\Omega} = \frac{(\mathbf{r}-\mathbf{r}')\mathrm{d}\mathcal{S}'}{|\mathbf{r}-\mathbf{r}'|^3}$$

- H_d does not depend on the size of the body (long-range interaction)
- Forces and torques scale with $1/\mathrm{size}$

Applications

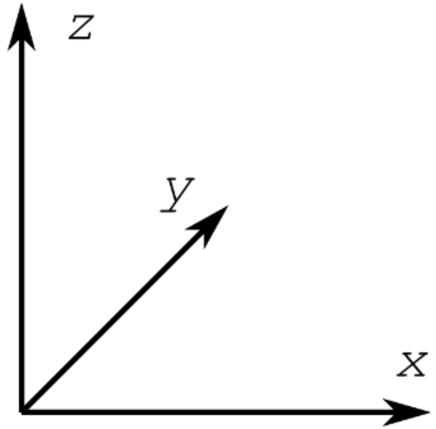
Demagnetizing tensor

$$\langle\mathbf{H}_\mathrm{d}(\mathbf{r})\rangle = -M_\mathrm{s}\,\bar{\bar{\mathbf{N}}}\cdot\mathbf{m}$$

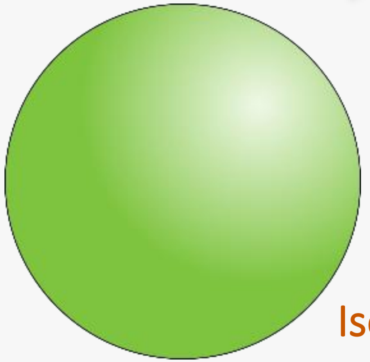
Applies to uniform magnetization

- Along main directions $\langle H_{\mathrm{d},i}(\mathbf{r})\rangle = -N_i M_\mathrm{s}$

Demagnetizing coefficient (examples)



Sphere $L_x = L_y = L_z = D$

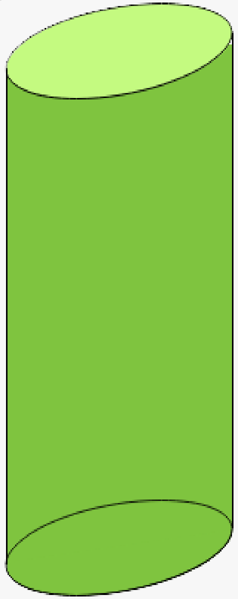


$N_x = N_y = N_z = \frac{1}{3}$

Isotropic

Cylinder $L_x = L_y = D$

$L_z = \infty$



$N_x = N_y = \frac{1}{2}$

$N_z = 0$

Favors **axial magnetization**

Slab (thin film) $L_x = L_y = \infty$

$N_x = N_y = 0$

$N_z = 1$

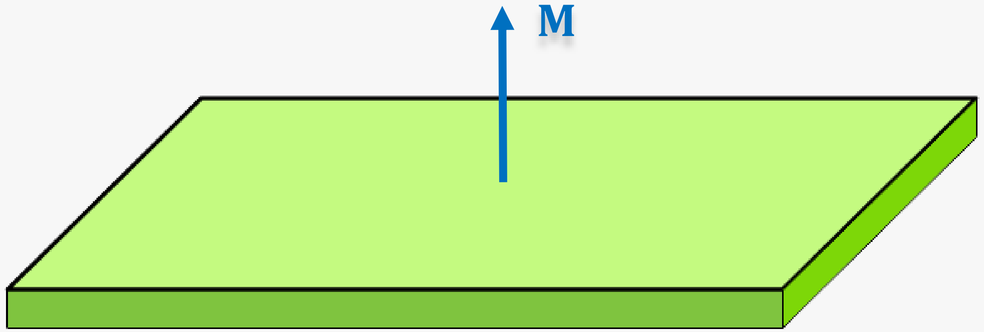
Favors in-plane magnetization

Take-away message

Dipolar energy favors alignment of magnetization with longest direction of sample

Quizz 1 – Stray field above thin film

What is the value of stray field above an extended perpendicularly-magnetized thin film?

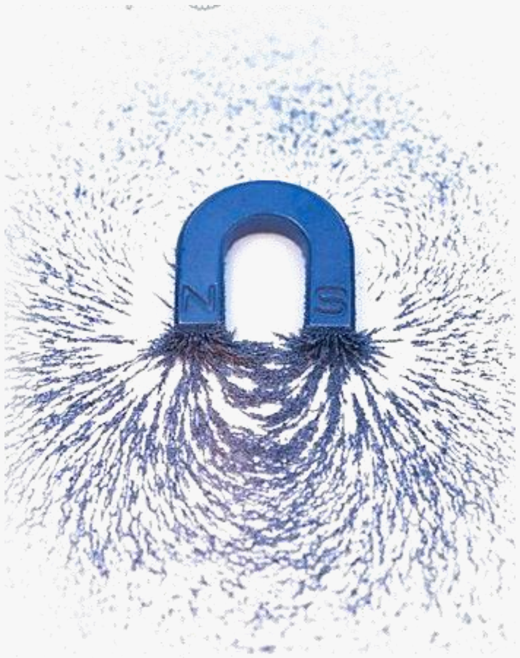


1. Zero
2. + M / 2
3. + M
4. Depends on the value of anisotropy

Quizz 2 – Maximum stray field

What is the maximum stray field (in free space) that can be obtained from a uniformly-magnetized system of arbitrary shape?

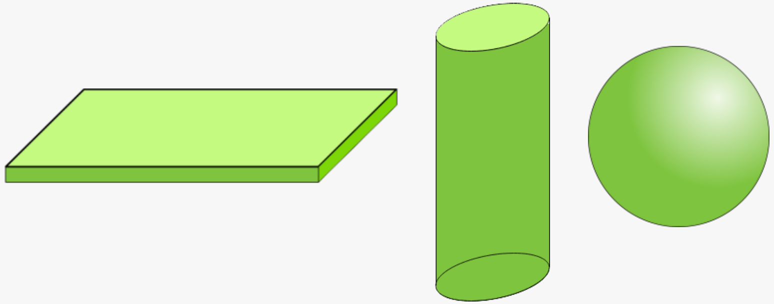
1. M /3
2. M /2
3. M
4. As high as we like, no limit



Quizz 3 – Demagnetizing coefficients

One can define and compute (analytics or numerics) demagnetizing coefficients for…

1. Ellipsoids, slabs, cylinders



2. Any finite-size body made up of one single piece

3. Any finite-size body of arbitrary shape

LA PANTHÈRE ROSE

4. None, all is an approximation

Magnetocrystaline anisotropy – Bulk

spintec SPIN IN ELECTRONICS

Underlying physics

- Crystal electric field (CEF): Coulomb interaction between electronic orbitals and the crystal environment $\mathcal{H}_{\mathrm{CEF}}$
- Spin-orbit coupling S and L $\mathcal{H}_{\mathrm{SO}}$

	$\mathcal{H}_{\mathrm{CEF}}$	$\mathcal{H}_{\mathrm{SO}}$
3d	$1-10\ \mathrm{eV}$	$10-100\ \mathrm{meV}$
4f	$25\ \mathrm{meV}$	$100-500\ \mathrm{meV}$

Phenomenology

- Angular dependence of the energy of a magnetic material
- Applies to all orders: ferromagnets, antiferromagnets etc.
- Group theory predict terms in expansions:

Cubic $E_{\mathrm{mc}} = K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^3 + \alpha_3^2\alpha_1^2) + K_2\alpha_1^2\alpha_2^2\alpha_3^2 + \cdots$

Hexagonal

$$E_{\mathrm{mc}} = K_1 \sin^2\theta + K_2 \sin^4\theta + K_3 \sin^6\theta + K_3' \sin^6\theta \sin^6\phi + \cdots$$

Numbers

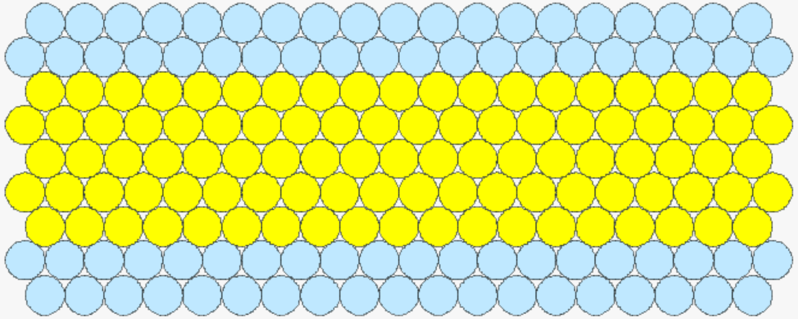
- Low symmetry favors high anisotropy
- Large range of values in known materials

Crucial importance for applications

- Compass, spintronic-based magnetic sensors
- Magnetic recording, including tapes, hard-disk drives, magnetic random access memories

Interfacial magnetic anisotropy

Simple picture: interfacial magnetic anisotropy



- Breaking of symmetry for surface/interface atoms
- Brings a correction to magnetocrystalline anisotropy

$$E_s = K_{s,1}\cos^2\theta + K_{s,2}\cos^4\theta + \cdots$$

L. Néel, J. Phys. Radium 15, 15 (1954),
Superficial magnetic anisotropy and orientational superstructures

This surface energy, of the order of 0.1 to 1 erg/cm2, is liable to play a significant role in the properties of ferromagnetic materials spread in elements of dimensions smaller than 100Å.

Visionary !!!

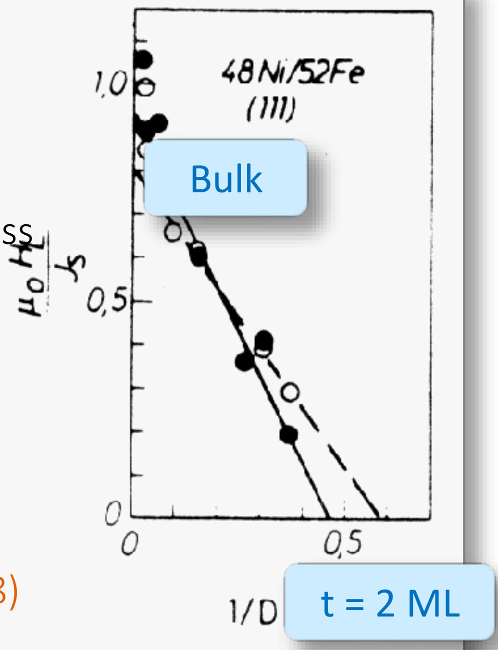
First Experimental evidence

- Total anisotropy energy

$$\mathcal{E}(t) = K_V t + 2K_s$$

- Anisotropy per unit thickness

$$E(t) = K_V + \frac{2K_s}{t}$$



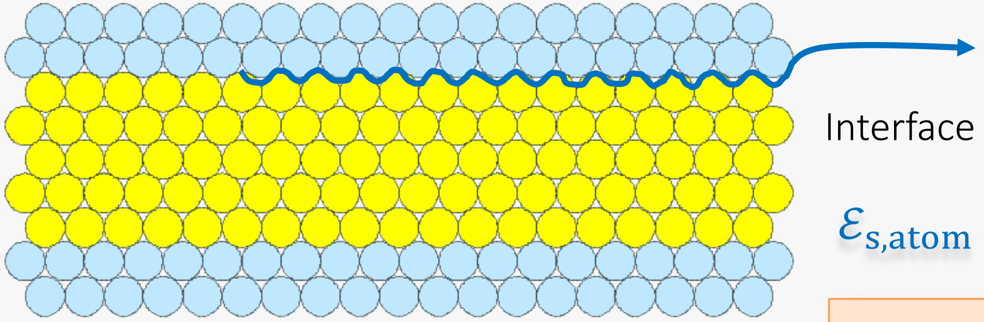
U. Gradmann and J. Müller,
Phys. Status Solidi 27, 313 (1968)

Figures

- Magnitude around $1\ \mathrm{mJ/m^2}$
- May promote perpendicular magnetization below thickness 1nm or so
- 80's & 90's: in contact with high spin-orbit materials (Au, Pt …)
- Since: Al_2O_3, MgO, graphene…

Atomic contribution to magnetostatic energy

Surface contribution to shape anisotropy



Interface dipolar anisotropy (per interface)

$$\mathcal{E}_{\mathrm{s,atom}} = -\mathrm{k_s}\delta\left(\frac{1}{2}\mu_0 M_s^2\right)\cos^2(\theta)$$

- Decreases slab demag coeff 1
- Change is large for open surface

Surface	k_s
fcc(111)	0.0344
fcc(001)	0.1178
hcc(110)	0.0383
bcc(001)	0.2187
hcp(0001)	0.0338

H.J.G. Draaisma et al., JAP 64, 3610 (1988)

Notes

- Included in Néel's pair interaction model from 1954 !
- Specific case of the dipolar crystalline anisotropy

P. Bruno, Physical origins and theoretical models of magnetic anisotropy, Ferienkurse des Forschungszentrums Jülich, Ch.24 (1993)

- This effect is expected to be large in VdW materials, due to the large difference of in-plane versus out-of-plane distance between magnetic ions

Magneto-elastic anisotropy – Bulk

Phenomenology

- Dependence of magnetic anisotropy on strain
- Can be viewed as the strain-derive of magneto-crystalline anisotropy
- Source of
  - Magnetostriction: direction of magnetization induces strain
  - Inverse magnetostriction: strain tends to orient magnetization along specific directions
- Example: polycrystalline sample under uniaxial strain

$$E_{\mathrm{mel}} = -\lambda_S \frac{E}{2}(3\cos^2\theta - 1)\epsilon - \frac{1}{2}E\epsilon^2 + \cdots$$

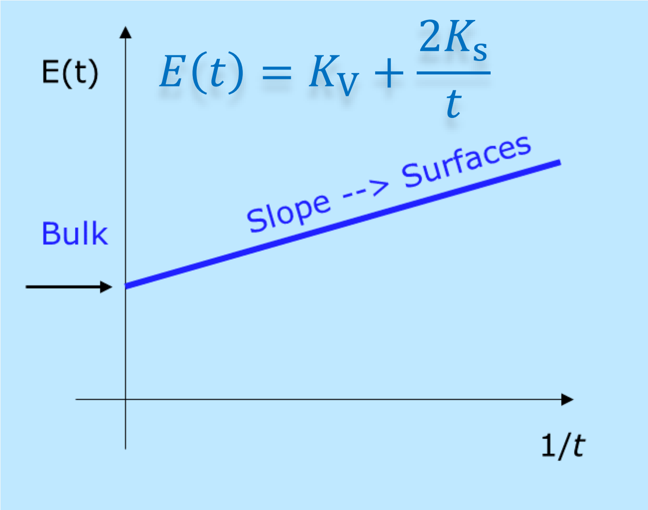
E Young modulus

Impact

- **Order of magnitude** of Lambda: 10^{-6}
- **Contributes to coercivity** in low-anisotropy materials
- Underpins effects such as **Invar**
- Magnetostriction is used in **actuators**

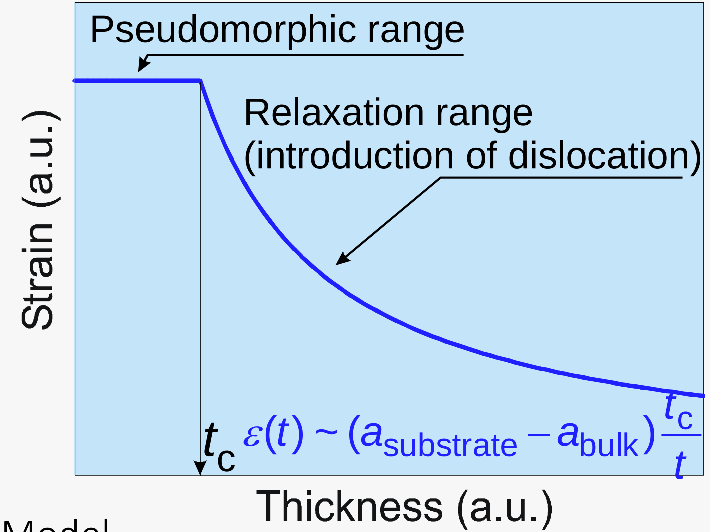
Magneto-elastic anisotropy – Thin films

Surface anisotropy alone



→ Bulk and surface anisotropy infered from intercept with *y* axis and slope

Inverse magnetostriction in films



Model

W. A. Jesser, Phys. Stat. Sol. 19, 95 (1967)

Experiments

U. Gradmann, Appl. Phys.3, 161 (1974)

→ Induces a 1/t variation of anisotropy, similar to interface contribution

C. Chappert and P. Bruno., JAP64, 5736 (1988)

Other effects…

- Interface roughness and intermixing
- Thickness-dependent thermal decay (when measured at T > OK)
- Non-linearity of magneto-elastic coupling coefficients

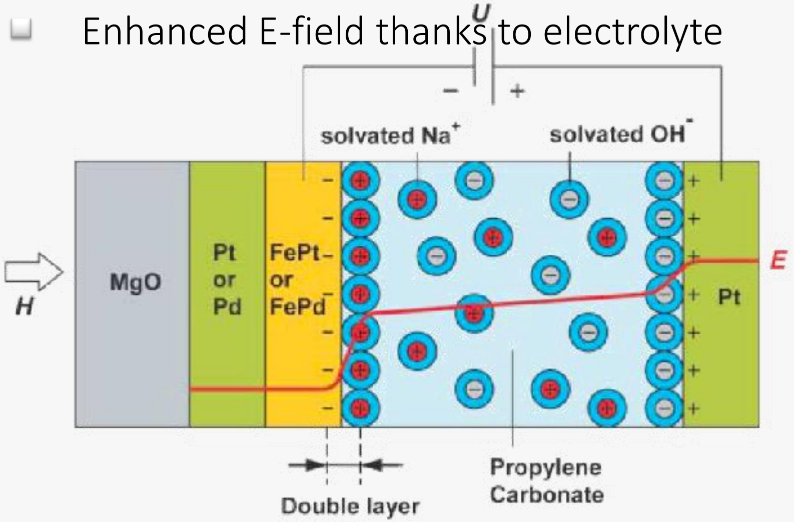
D. Sander, Rep. Prog. Phys. 62, 809 (1999)

U. Gradmann, Magnetism in ultrathin transition metal films, in Handbook of magnetic materials, vol. 7, Buschow, K. H. J. (Ed.), Elsevier (1993)

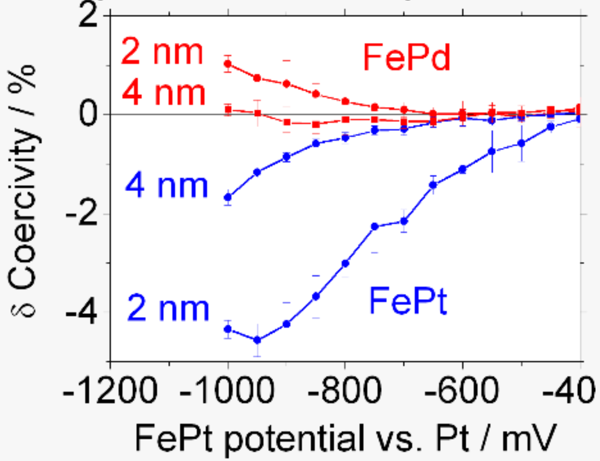
Voltage control of magnetization in metals

Seminal report

- Enhanced E-field thanks to electrolyte



- Slight (relative) change of coercivity



- Effect not expected for metals, due to short screening length
- Relative change of coercivity is weak as coercivity is large

M. Weisheit et al., Science 315, 349 (2007)

Developments

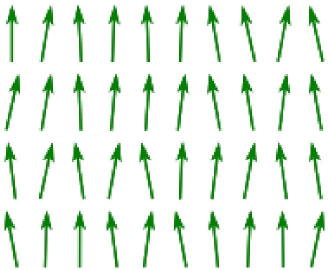
- Precessional switching with pulse of E-field
  Y. Shiota et al., Nature Mater.11, 39 (2012)
- Ferromagnetic resonance with ac E-field
  T. Nozaki et al., Nature Phys. 8, 491 (2012)
- Inversion of sign of DMI and skyrmions chirality
  R. Kumar et al., arXiv: 2009.13136 (2020)

Motivations for technology

- Drastically reduce Joule heating (only capacitance current)
- Gateable functionality

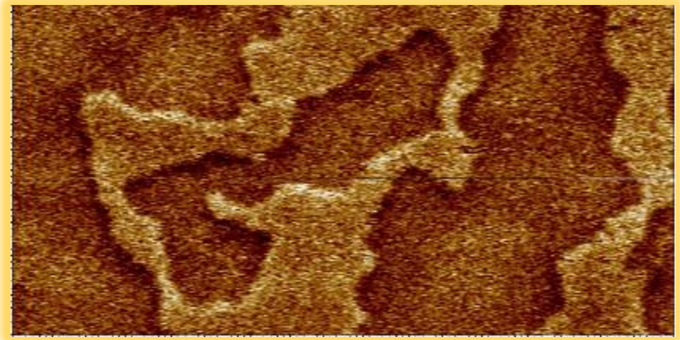
Table of contents

- Magnetic ordering



- Magnetic anisotropy

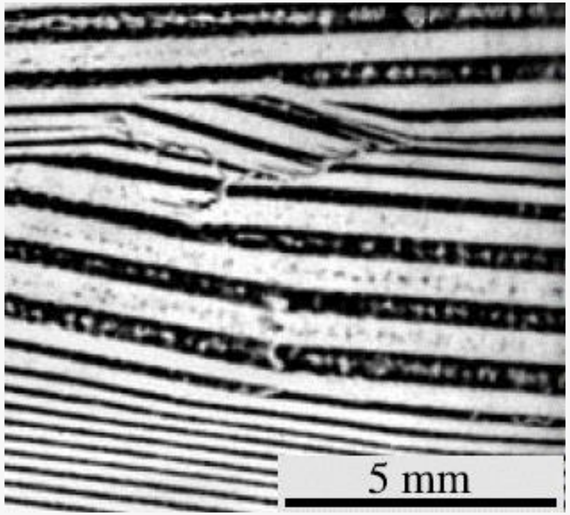
- Domains and magnetization processes



The origin of magnetic domains

Historical background

- **Puzzle from the early days** of magnetism: some materials may be magnetized under applied field, however "loose" their magnetization when the field is removed
- **Postulate from Weiss**: existence of magnetic domains, i.e., large (3D) regions with each uniform magnetization
- **Magnetic domain walls** are the narrow (2D: planes) regions separating neighboring domains

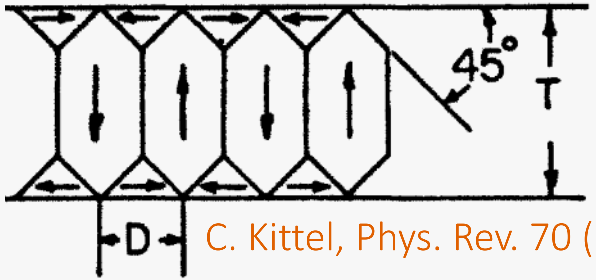


FeSi sheet (transformer)

A. Hubert, magnetic domains

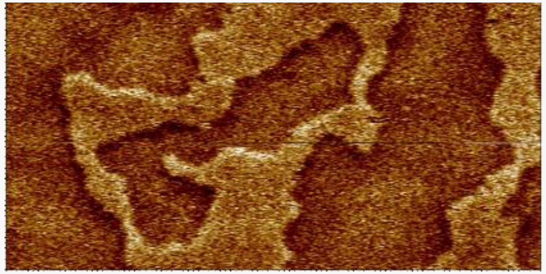
Origin of domains

- **Minimization of energy:** closure of magnetic flux to decrease dipolar energy, at the expense of energy in the domain walls (exchange, anisotropy…)



C. Kittel, Phys. Rev. 70 (11&12), 965 (1946)

- **Magnetic history:** magnetic domains along various directions may form through the ordering transition or following a partial magnetization process, persisting even though leaving the system not in the ground state



MgO\Co[1nm)\Pt

Magnetic Force Microscopy, 5 x 2.5 µm

Micromagnetism formalism

Magnetization

Magnetization vector **M**

- **Continuous** function
- May vary over time and space
- **Modulus is constant and uniform** (hypothesis in micromagnetism)

$$\mathbf{M}(\mathbf{r}) = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = M_s \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix}$$

$$m_x^2 + m_y^2 + m_z^2 = 1$$

Mean field approach is possible: $M_s = M_s(T)$

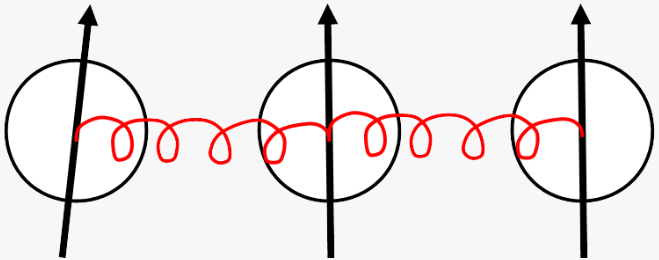
Exchange interaction

- Atomistic view $\mathcal{E} = -\sum_{i \neq j} J_{i,j} \mathbf{S}_i \cdot \mathbf{S}_j$ (total energy, J)
- Micromagnetic view $\mathbf{S}_i \cdot \mathbf{S}_j = S^2 \cos(\theta_{i,j}) \approx S^2 \left(1 - \frac{\theta_{i,j}^2}{2}\right)$

$$E_{\text{ex}} = A(\boldsymbol{\nabla} \cdot \mathrm{m})^2 = A \sum_{i,j} \left(\frac{\partial m_i}{\partial x_j}\right)^2$$

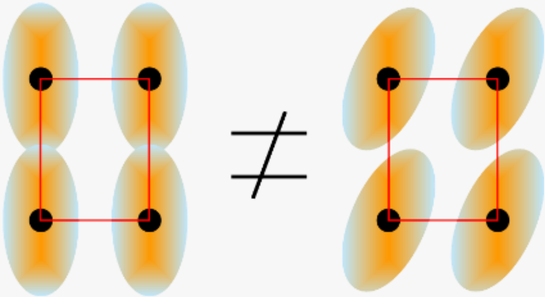
The various types of magnetic energy

Exchange energy



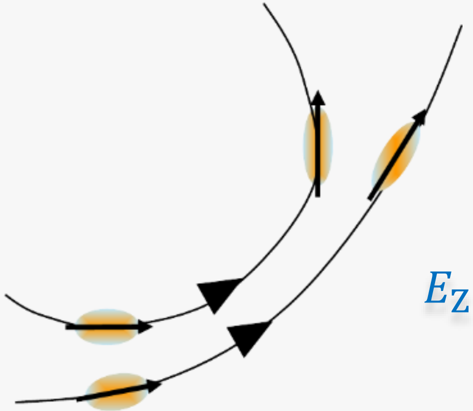
$$E_{\mathrm{ex}} = A(\boldsymbol{\nabla}\cdot\mathrm{m})^2 = A\sum_{i,j}\left(\frac{\partial m_i}{\partial x_j}\right)^2$$

Magnetocrystalline anisotropy energy



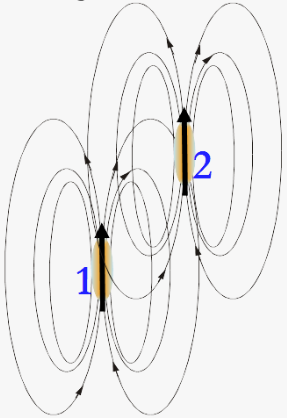
$$E_{\mathrm{mc}} = K\,f(\theta,\varphi)$$

Zeeman energy (→ enthalpy)



$$E_{\mathrm{Z}} = -\mu_0 \mathbf{M}.\mathbf{H}$$

Magnetostatic energy



$$E_{\mathrm{d}} = -\frac{1}{2}\mu_0 \mathbf{M}\cdot\mathbf{H}_{\mathrm{d}}$$

Magnetic length scales

The dipolar exchange length

When: anisotropy and exchange compete

$$E = A\left(\frac{\partial m_i}{\partial x_j}\right)^2 + K_\mathrm{d}\sin^2\theta$$

Exchange: $\mathrm{J/m}$

Dipolar: $\mathrm{J/m^3}$

$$K_\mathrm{d} = \frac{1}{2}\mu_0 M_\mathrm{s}^2$$

$$\Delta_\mathrm{d} = \sqrt{A/K_\mathrm{d}} = \sqrt{2A/\mu_0 M_\mathrm{s}^2}$$

$$\Delta_\mathrm{d} \simeq 3 - 10\ \mathrm{nm}$$

Critical single-domain size, relevant for small particles made of soft magnetic materials

Often called: exchange length

The anisotropy exchange length

When: anisotropy and exchange compete

$$E = A\left(\frac{\partial m_i}{\partial x_j}\right)^2 + K\sin^2\theta$$

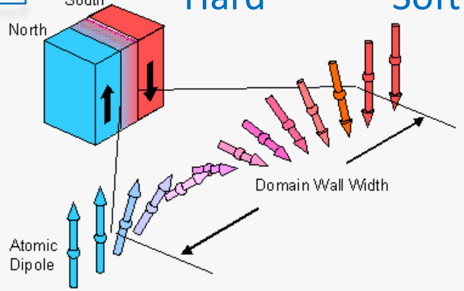
Exchange: $\mathrm{J/m}$

Anisotropy: $\mathrm{J/m^3}$

$$\Delta_\mathrm{u} = \sqrt{A/K}$$

$\Delta_\mathrm{u} \simeq 1\ \mathrm{nm} \rightarrow 100\ \mathrm{nm}$

Hard → Soft

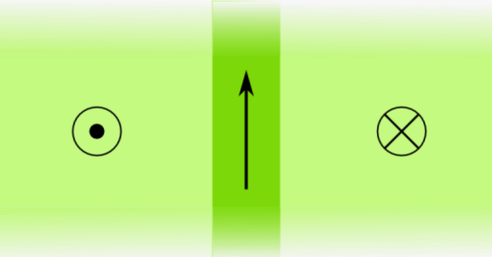


Sometimes called: Bloch parameter, or wall width

Note: Other length scales can be defined, e.g. with magnetic field

Magnetic domains walls (and dimensionality)

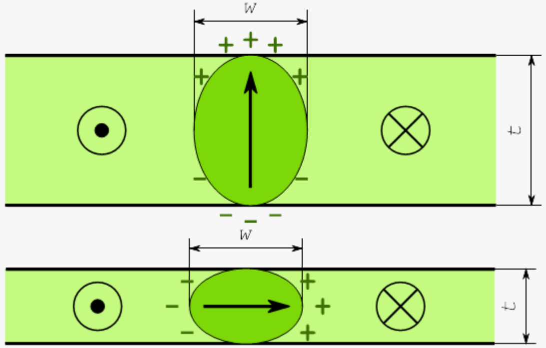
Bloch wall in the bulk (2D)



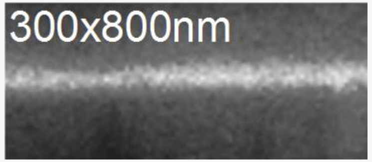
- No magnetostatic energy
- Width $\Delta_u = \sqrt{A/K}$
- Energy $\gamma_W = 4\sqrt{AK}$

F. Bloch, Z. Phys. 74, 295 (1932)

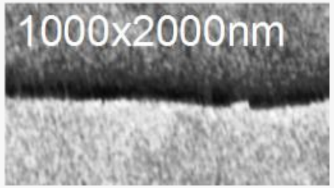
Domain walls in thin films (towards 1D)



Bloch wall
$t \gtrsim w$



Néel wall
$t \lesssim w$

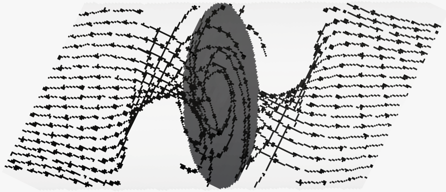


- Implies magnetostatic energy
- No exact analytic solution

L. Néel, C. R. Acad. Sciences 241, 533 (1956)

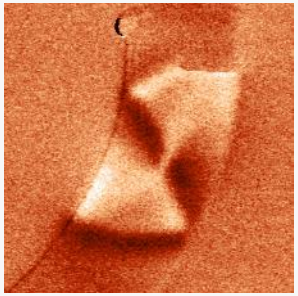
Bloch point (0D)

- Point with vanishing magnetization



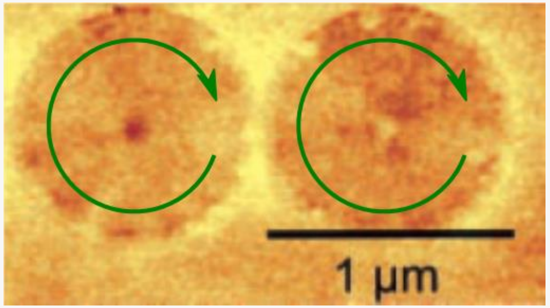
W. Döring,
JAP 39, 1006 (1968)

Constrained walls (eg in strips)



Permalloy (15nm)
Strip width 500nm

Vortex (1D → 0D)

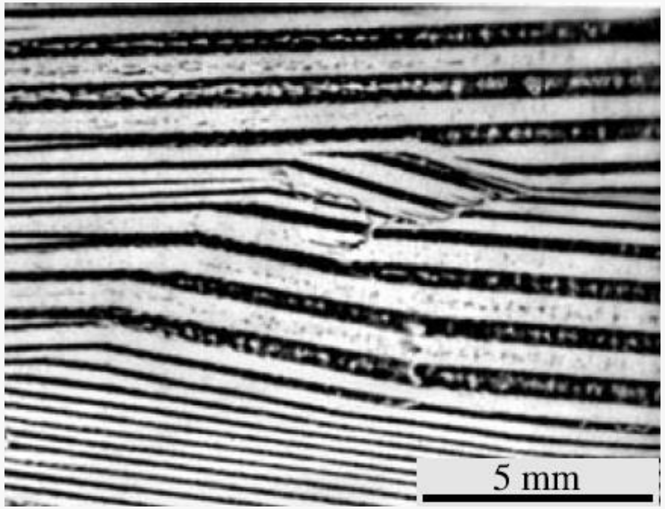


T. Shinjo et al.,
Science 289,
930 (2000)

Magnetic domains (and dimensionality)

Bulk materials

Numerous and complex magnetic domains

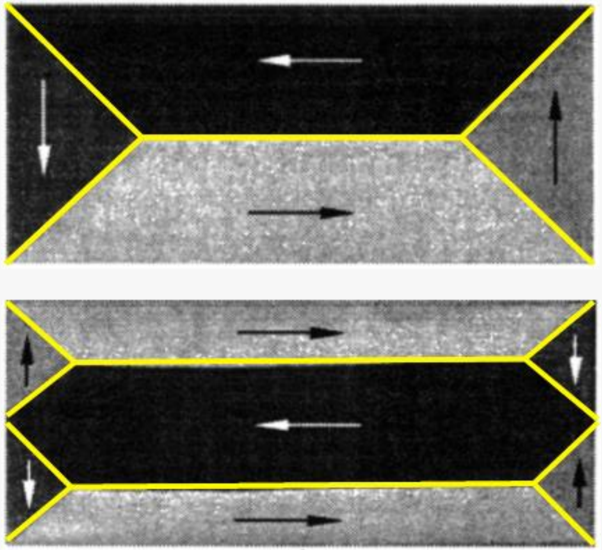


FeSi sheet (transformer)

A. Hubert, magnetic domains

Mesoscopic scale

Small number of domains, simple shape

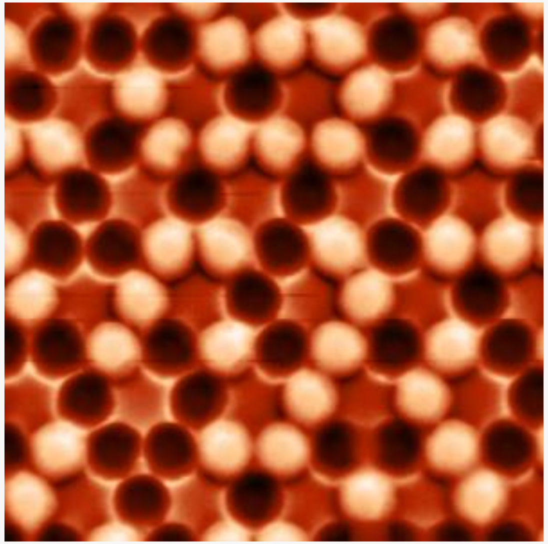


Microfabricated dots, Kerr magnetic imaging

A. Hubert, magnetic domains

Nanometric scale

Magnetic single domain



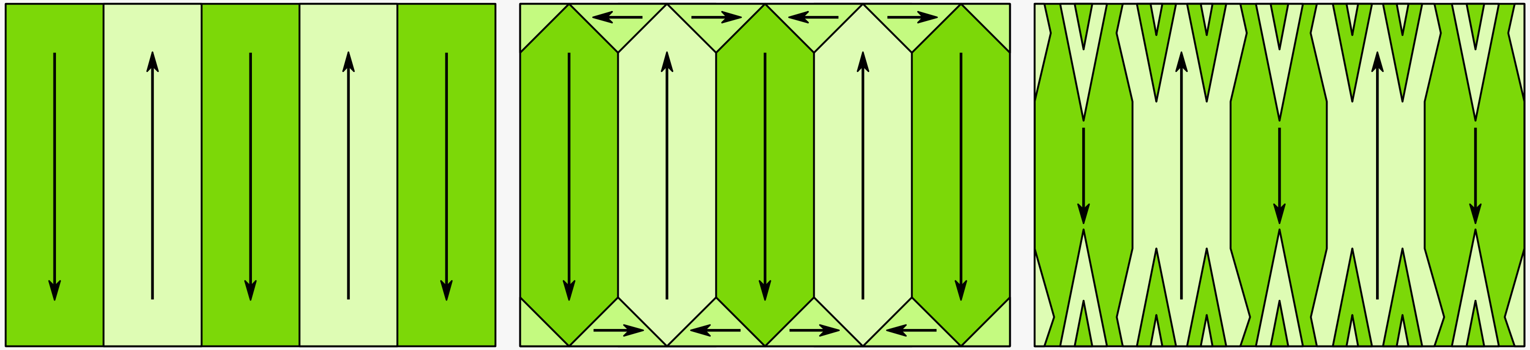
Microfabricated dots, magnetic force microscopy

Sample courtesy: I. Chioar

Statics – Tendency for flux-closure domains

Films with easy axis out-of-the-plane: Kittel domains

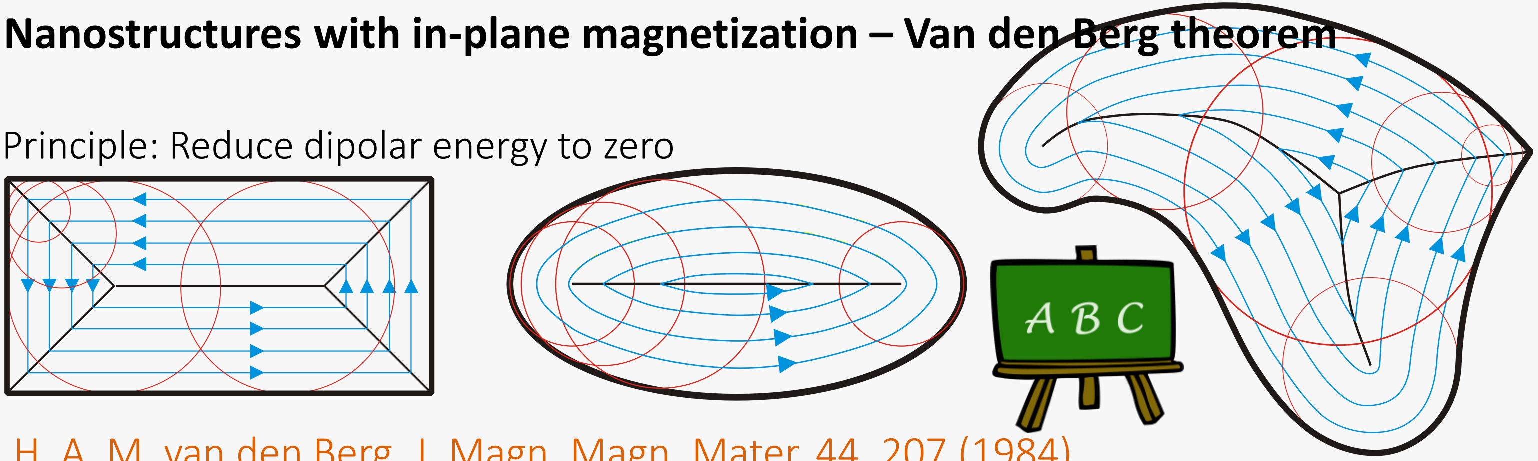
Principle: compromise between gain in dipolar energy, and cost in wall energy



C. Kittel, Physical theory of ferromagnetic domains, Rev. Mod. Phys. 21, 541 (1949)

Nanostructures with in-plane magnetization – Van den Berg theorem

Principle: Reduce dipolar energy to zero



H. A. M. van den Berg, J. Magn. Magn. Mater. 44, 207 (1984)

Magnetostatics – End domains and curling

Historical background

Introduced in the context of the Brown paradox for magnetization reversal

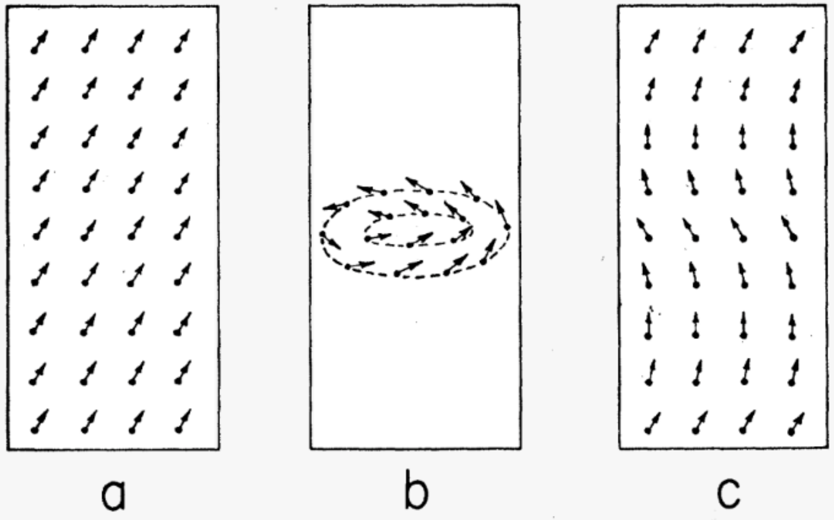
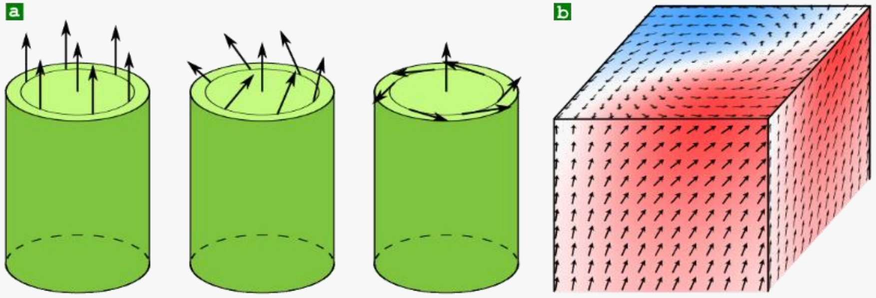


FIG. 2. Modes of magnetization change for the infinite cylinder: (a) spin rotation in unison; (b) magnetization curling; (c) magnetization buckling.

E. H. Frei, Phys. Rev. 106, 446 (1957)

Example in 3D nanomagnets

End curling in elongated 3D objects (wires etc.)



Curling spreads surface charges into volume charges

$$\sigma(\mathbf{r}) = M_s\,\mathbf{m}(\mathbf{r})\cdot\mathbf{n}(\mathbf{r})$$

$$\rho(\mathbf{r}) = -M_s\,\boldsymbol{\nabla}\cdot\mathbf{m}(\mathbf{r}) = -M_s\frac{\partial m_z}{\partial z}$$

Notes

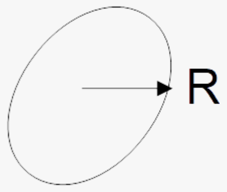
- Surface + volume charges is conserved
- Curling may develop whenever a dimension is larger than 7 dipolar exchange lengths $\Delta_d = \sqrt{2A/\mu_0 M_s^2}$

Magnetostatics – Range considerations

Range

Example: upper bound of dipolar field in thin films

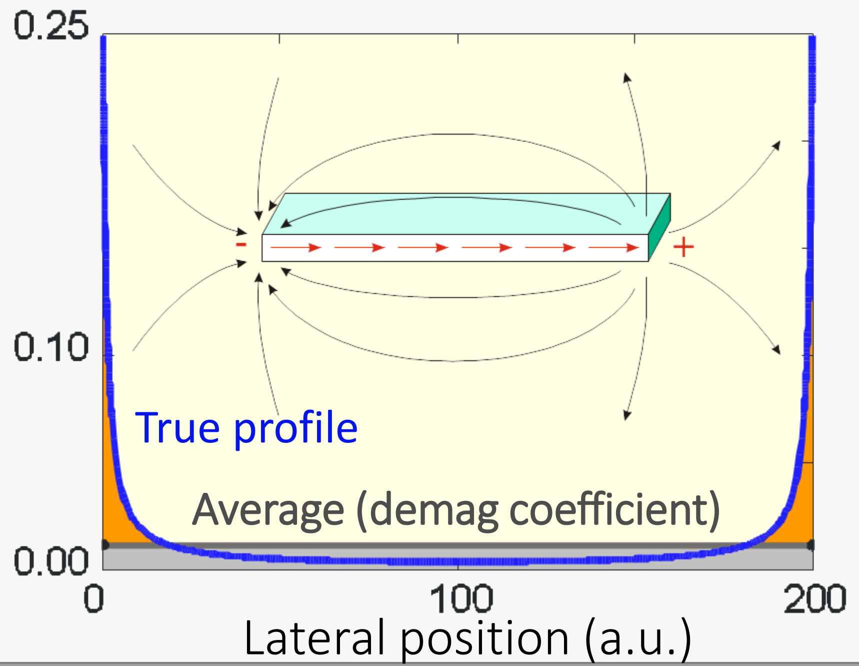
$$\|\mathbf{H}_\mathrm{d}(\mathbf{r})\| \leq M_\mathrm{s} t \int \frac{2\pi r}{r^3}\,\mathrm{d}r$$



$$\|\mathbf{H}_\mathrm{d}(\mathbf{R})\| \leq \mathrm{Cste} + \mathcal{O}(1/R)$$

Non-homogeneity

Example: flat strip with aspect ratio 0.0125



- Dipolar fields are short-ranged and inhomogeneous in low dimensions
- Consequences: non-uniform magnetization switching, edge modes etc.

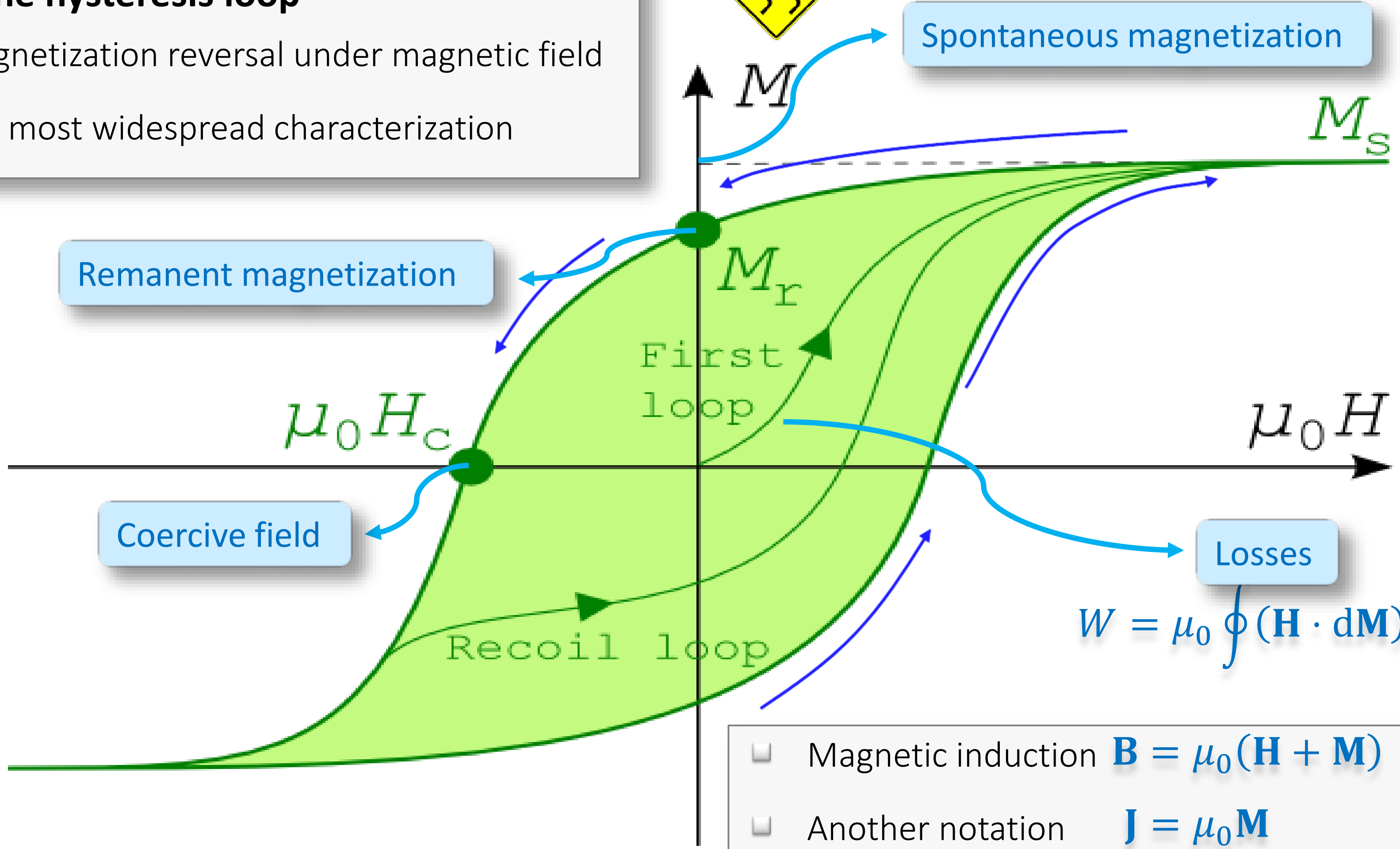
➔ A 1D/2D system in space behaves very differently from a nano-bulk magnet

The hysteresis loop

The hysteresis loop

- Magnetization reversal under magnetic field
- The most widespread characterization

Spontaneous ≠ Saturation



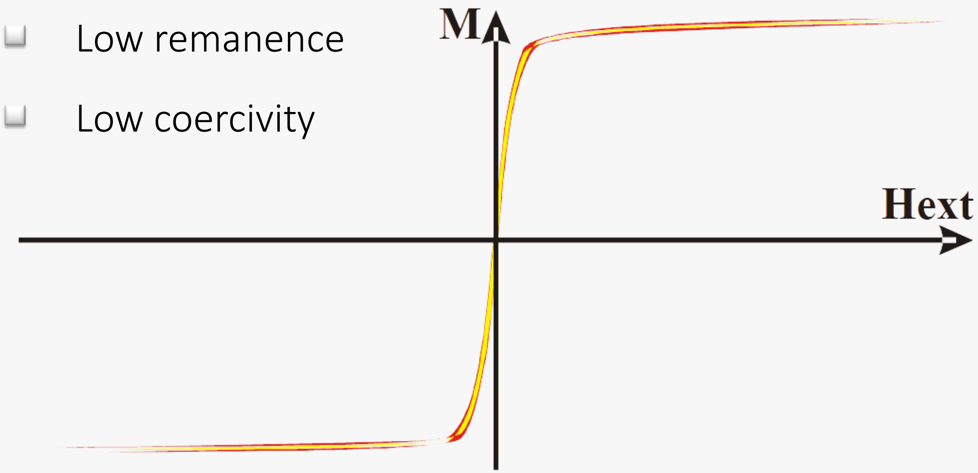
$$W = \mu_0 \oint (\mathbf{H} \cdot \mathrm{d}\mathbf{M})$$

- Magnetic induction $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$
- Another notation $\mathbf{J} = \mu_0 \mathbf{M}$

The hysteresis loop

Soft-magnetic materials

- Low remanence
- Low coercivity

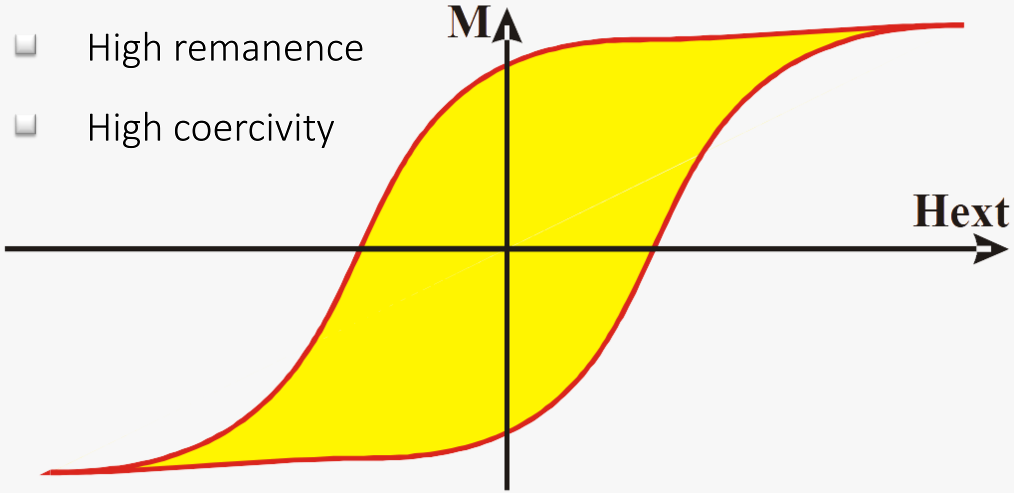


Applications

- Transformers
- Flux guides, sensors
- Magnetic shielding

Hard-magnetic materials

- High remanence
- High coercivity



Applications

- Permanent magnets,
- Motors and generators
- Magnetic recording

The Stoner-Wohlfarth model

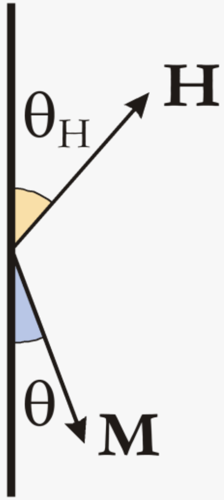
Framework: uniform magnetization

- Drastic, unsuitable in most cases
- Remember: demagnetization field may not be uniform

$$\mathcal{E} = E\mathcal{V}$$

$$= \mathcal{V}[K_{\mathrm{eff}} \sin^2\theta - \mu_0 M_s H \cos(\theta - \theta_H)]$$

- Anisotropy: $K_{\mathrm{eff}} = K_{\mathrm{mc}} + (\Delta N) K_{\mathrm{d}}$



L. Néel, Compte rendu Acad. Sciences 224, 1550 (1947)

E. C. Stoner and E. P. Wohlfarth,

Phil. Trans. Royal. Soc. London A240, 599 (1948)

Reprint: IEEE Trans. Magn. 27(4), 3469 (1991)

Names used

- Uniform rotation / magnetization reversal
- Coherent rotation / magnetization reversal
- Macrospin etc.

Dimensionless units

$$e = \sin^2\theta - 2h\cos(\theta - \theta_H)$$

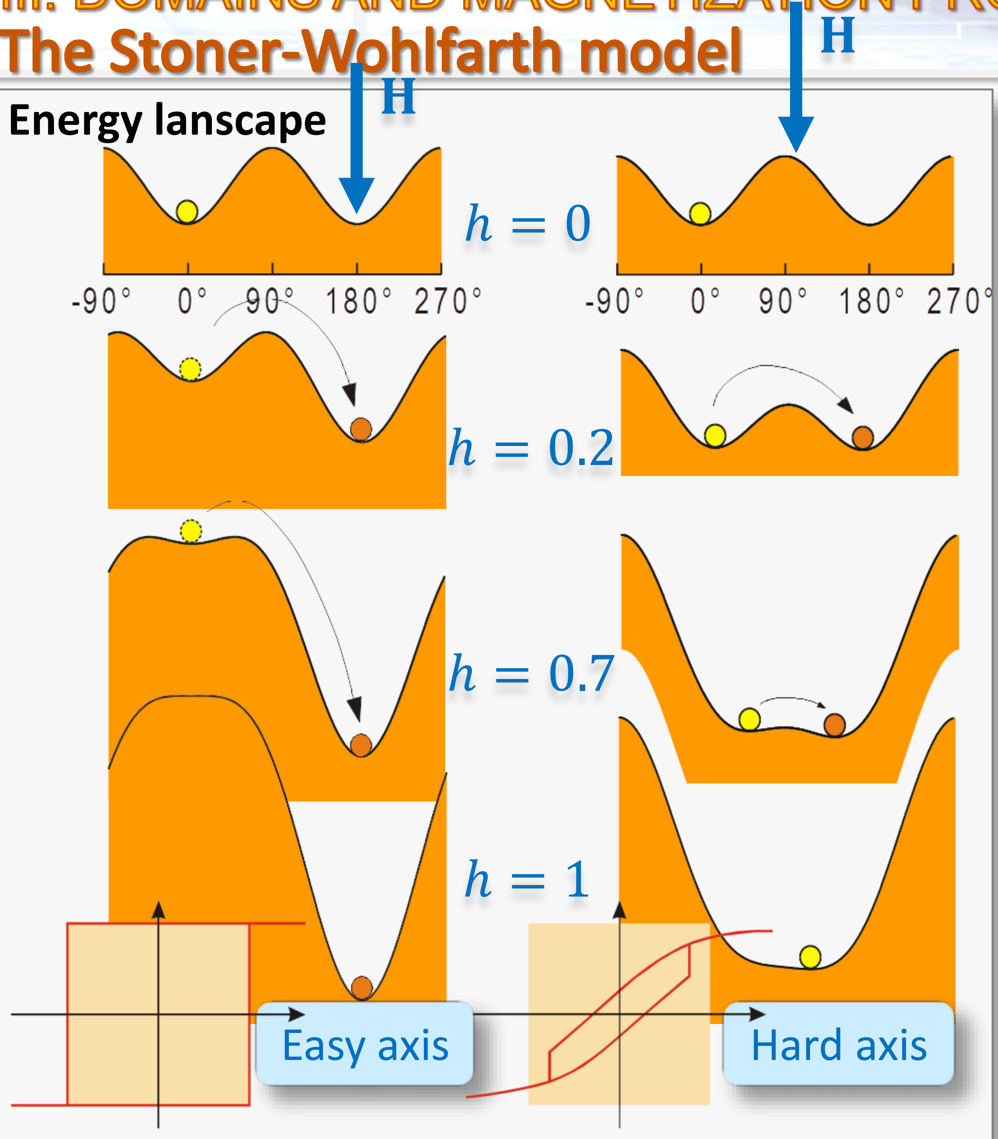
$$e = \mathcal{E}/(K\mathcal{V})$$

$$h = H/H_{\mathrm{a}}$$

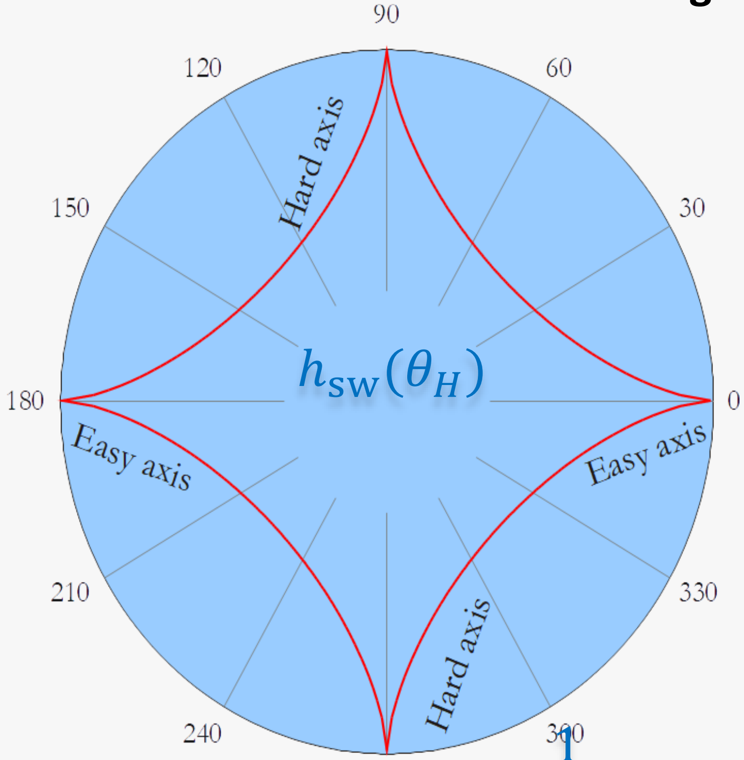
$$H_{\mathrm{a}} = 2K/(\mu_0 M_s)$$

The Stoner-Wohlfarth model

Energy lanscape



Stoner-Wohlfarth astroid: switching field



$$h_{sw}(\theta_H) = \frac{1}{(\sin^{2/3}\theta_H + \cos^{2/3}\theta_H)^{3/2}}$$

J. C. Slonczewski, Research Memo RM 003.111.224, IBM Research Center (1956)

Macrospins – Switching field versus coercive field

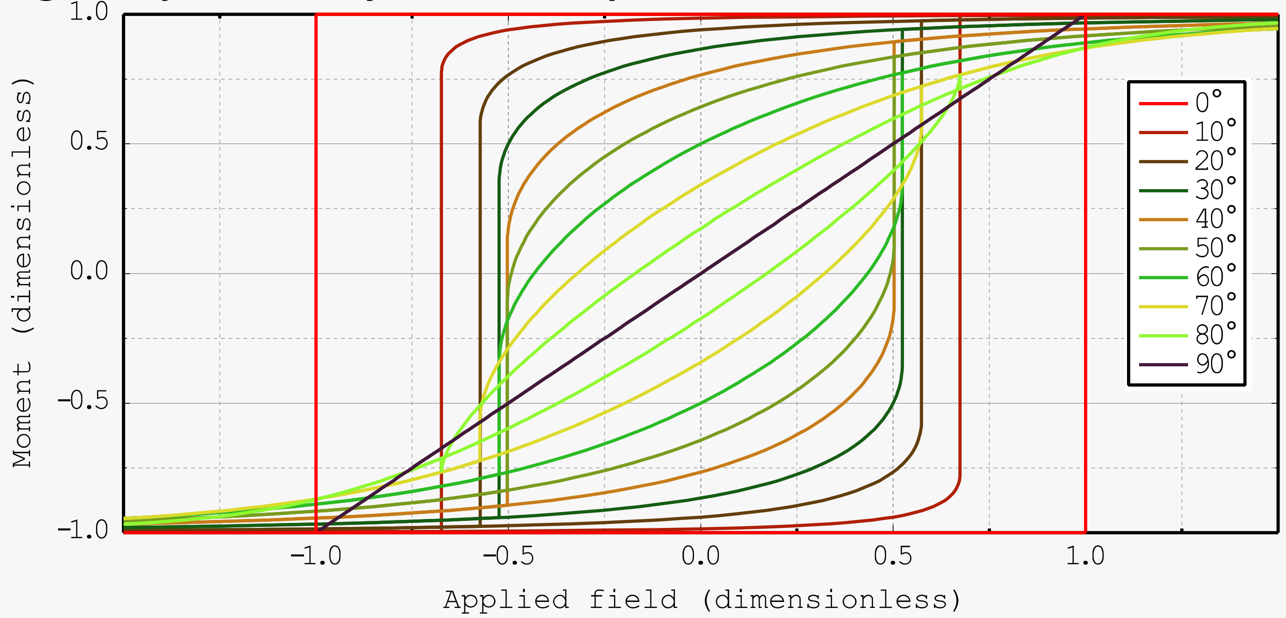
Switching field H_{sw}

- A value of field at which an irreversible (abrupt) jump of magnetization angle occurs.
- Can be measured only in single particles.

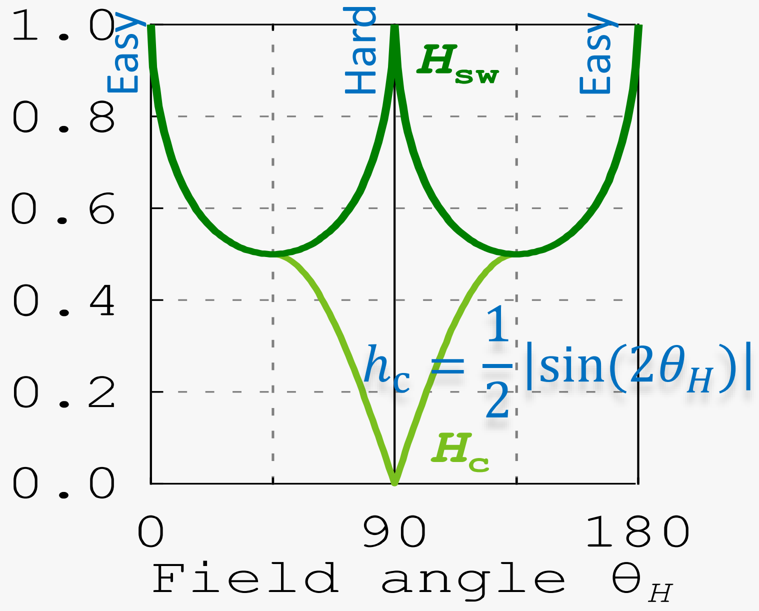
Coercive field H_{c}

- The field at which $\mathbf{H}\cdot\mathbf{M}=0$
- Measurable in materials (large number of 'particles').
- May or may not be a measure of the mean switching field at the microscopic level

Angle-dependent hysteresis loops

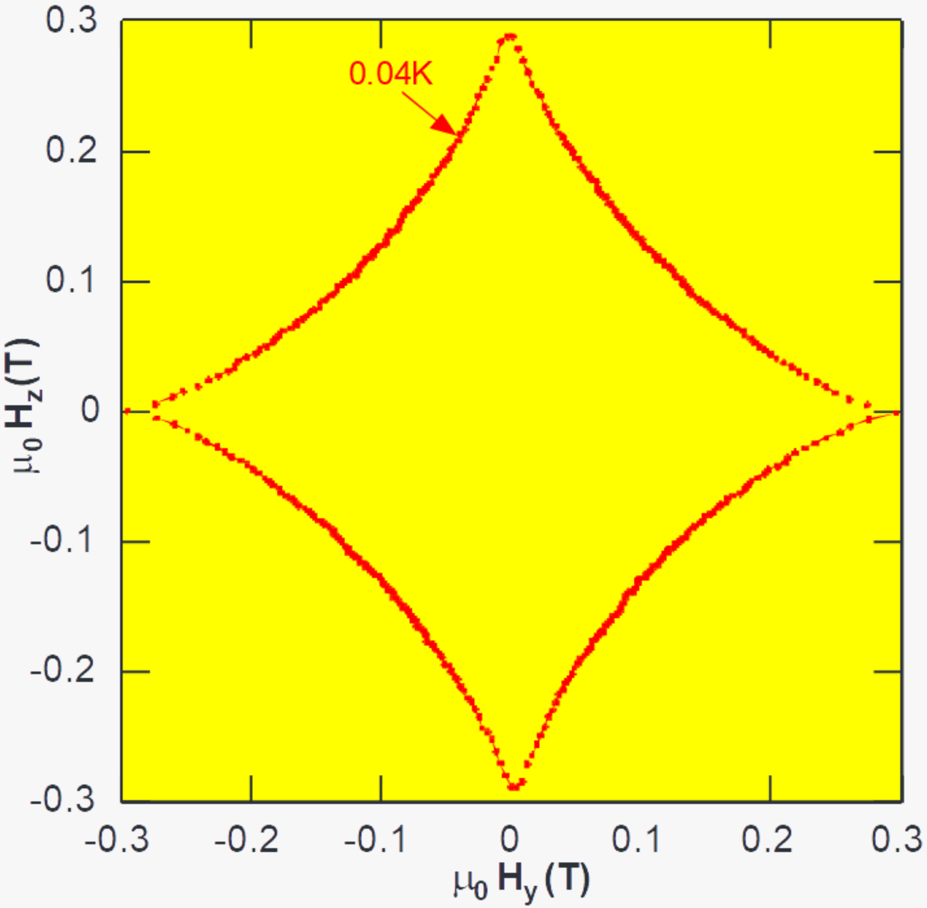


Switching versus coercive field

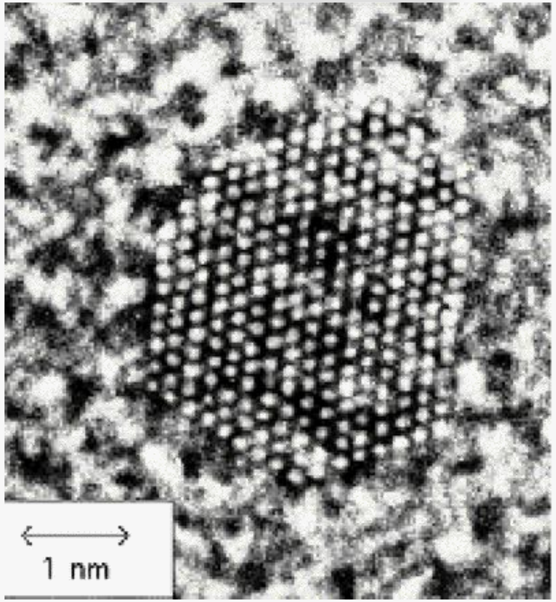


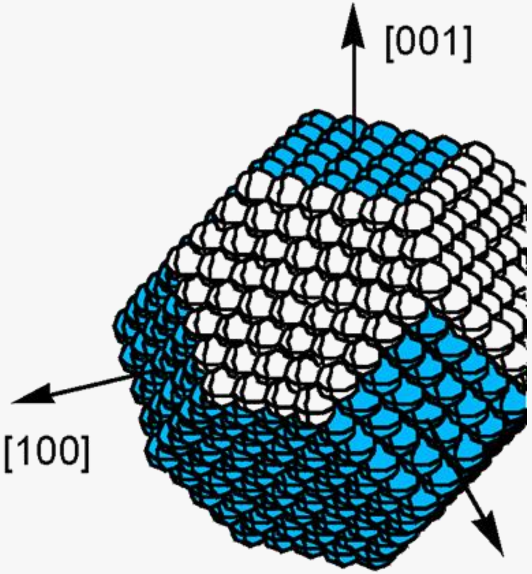
Macrospins – Experiments

First experimental evidence



Co cluster

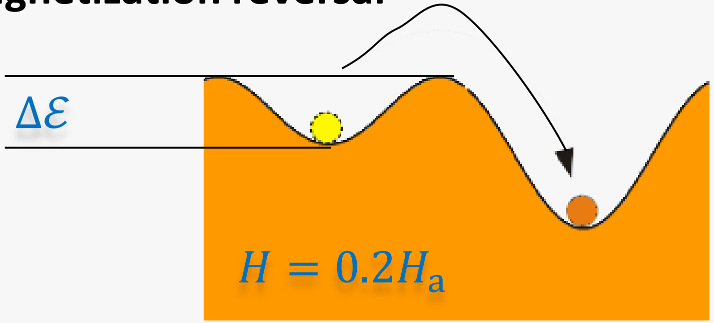




W. Wernsdorfer et al., Phys. Rev. Lett. 78, 1791 (1997)

Superparamagnetism and the blocking temperature

Energy barrier preventing magnetization reversal



$$\Delta\mathcal{E} = K\mathcal{V}\left(1 - \frac{H}{H_a}\right)^2$$

E. F. Kneller, J. Wijn (ed.) Handbuch der Physik XIII/2: Ferromagnetismus, Springer, 438 (1966)

M. P. Sharrock, J. Appl. Phys. 76, 6413-6418 (1994)

Thermal activation

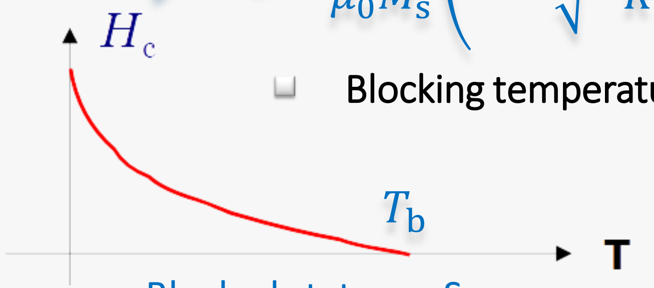
Brown, Phys.Rev.130, 1677 (1963)

- **Waiting time** (Arrhenius law) $\tau = \tau_0 \exp\left(\frac{\Delta\mathcal{E}}{k_B T}\right)$

  ➡ $\Delta\mathcal{E} = k_B T \ln\left(\frac{\tau}{\tau_0}\right)$

- Lab measurement: $\tau \approx 1\ \mathrm{s}$ ➡ $\Delta\mathcal{E} \approx 25 k_B T$

  ➡ $H_c = \frac{2K}{\mu_0 M_s}\left(1 - \sqrt{\frac{25 k_B T}{K\mathcal{V}}}\right)$



- Blocking temperature: $T_b \approx \frac{K\mathcal{V}}{25 k_B}$

Blocked state Superparamagnetic state

- Coercivity and remanence are lost at small size
- Incentive to enhance magnetic anisotropy

The case of magnetic recording or memory

$\tau \approx 10^9\ \mathrm{s}$ ➡ $K\mathcal{V}_b \approx 40-60\, k_B T$

Superparamagnetism – Modeling

Formalism

- Energy

$$\mathcal{E} = K\mathcal{V}\mathrm{f}(\theta,\phi) - \mu_0\mu H$$

- Partition function

$$Z = \sum \exp(-\beta\mathcal{E})$$

- Average moment

$$\langle\mu\rangle = \frac{1}{\beta\mu_0 Z}\frac{\partial Z}{\partial H}$$

Isotropic case

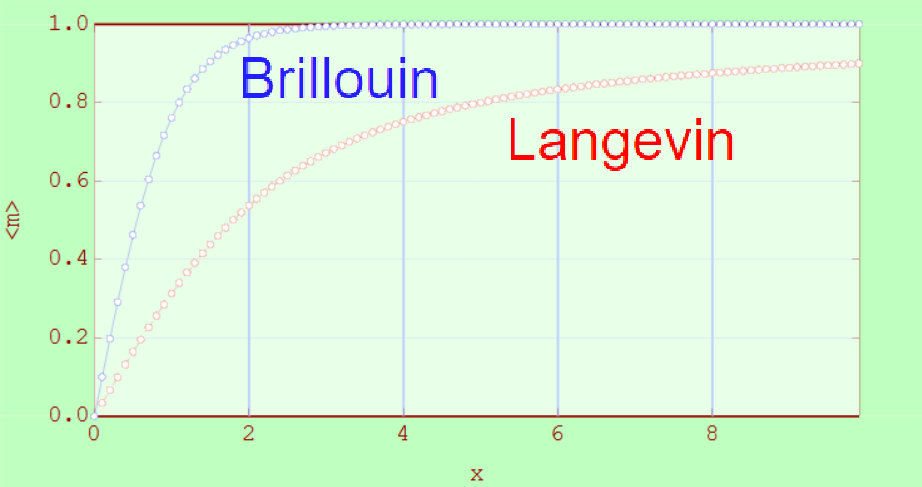
$$Z = \int_{-\mathcal{M}}^{\mathcal{M}} \exp(-\beta\mathcal{E})\,\mathrm{d}\mu$$

Note: equivalent to integrate on solid angle

$$\langle\mu\rangle = \mathcal{M}\left[\coth(x) - \frac{1}{x}\right]$$

Langevin function

Note: refers to the moment of the particle, not a spin ½



Highly anisotropic case

$$Z = \exp(\beta\mu_0\mathcal{M}H) + \exp(-\beta\mu_0\mathcal{M}H)$$

Note: only two states are populated, 'up' and 'down'

$$\langle\mu\rangle = \mathcal{M}\tanh(x)$$

Brillouin ½ function

- Fit M(H) curve to extract magnetization (and hence the volume) of nanoparticles
- Beware of anisotropy strength and distribution in fits !

Magnetization switching of extended systems

Brown paradox

In most (extended systems): $H_c \ll \dfrac{2K}{\mu_0 M_s}$

(Micromagnetic) modeling

Exhibit analytic, nevertheless realistic models for magnetization reversal

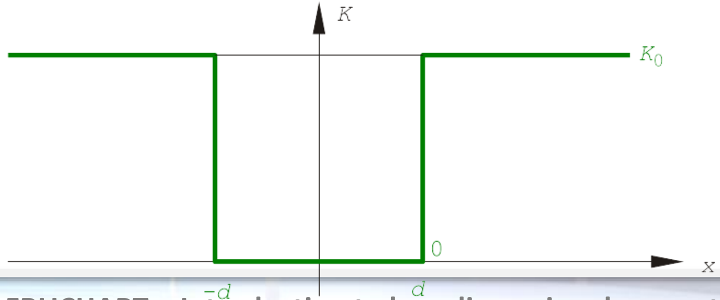
PHYSICAL REVIEW VOLUME 119, NUMBER 1 JULY 1, 1960

Reduction in Coercive Force Caused by a Certain Type of Imperfection

A. AHARONI
Department of Electronics, The Weizmann Institute of Science, Rehovot, Israel

(Received February 1, 1960)

As a first approach to the study of the dependence of the coercive force on imperfections in materials which have high magnetocrystalline anisotropy, the following one-dimensional model is treated. A material which is infinite in all directions has an infinite slab of finite width in which the anisotropy is 0. The coercive force is calculated as a function of the slab width. It is found that for relatively small widths there is a considerable reduction in the coercive force with respect to perfect material, but reduction saturates rapidly so that it is never by more than a factor of 4.



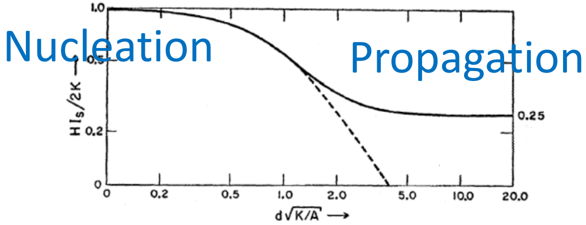
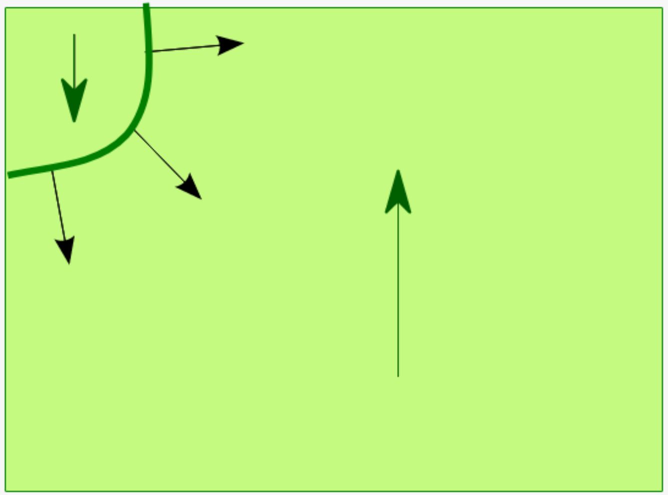


FIG. 1. The nucleation field (dashed) and coercive force (full curve) in terms of the coercive force of perfect material, $HI_s/2K$, as functions of the defect size, d.

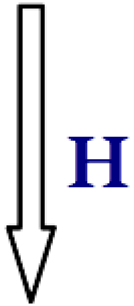
Nucleation – Propagation mechanisms

How are domain walls involved in magnetization reversal?

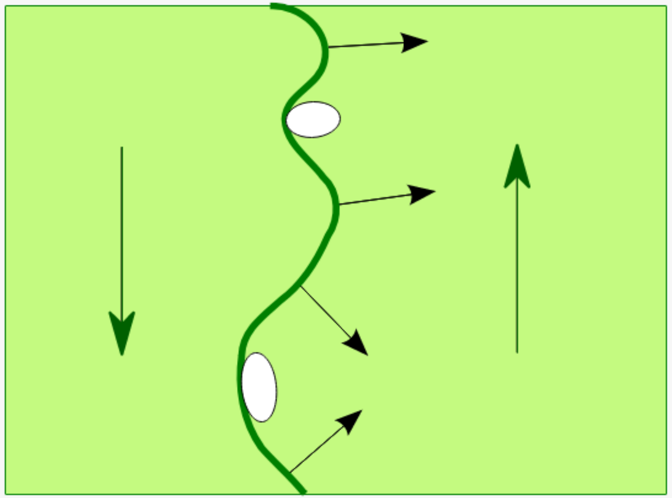
Coercivity determined by nucleation



- Concept of nucleation volume
- Physics has some similarity with that of the Stoner-Wohlfarth model for small particles



Coercivity determined by propagation

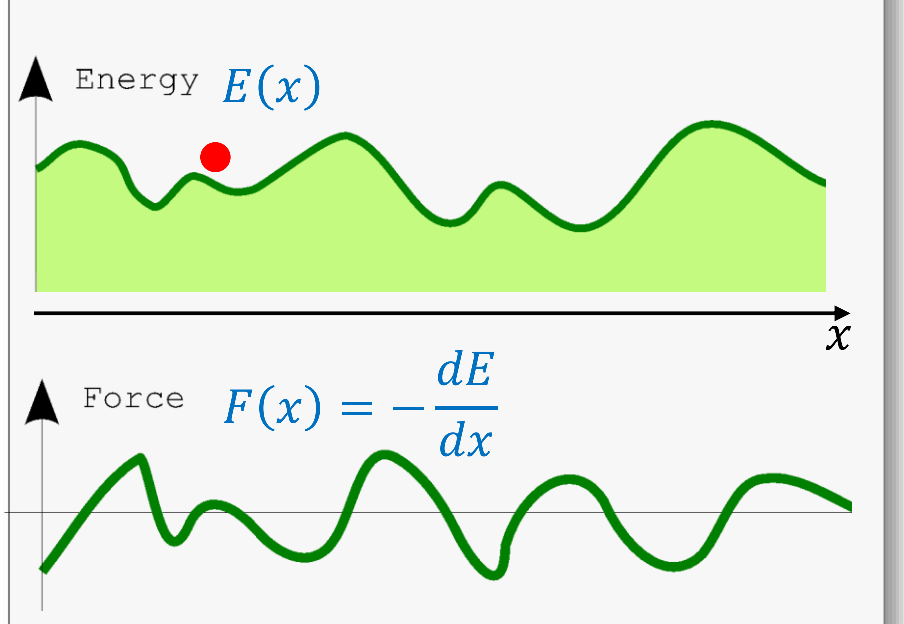


- Physics of surface/string in heterogeneous landscape
- Modeling necessary

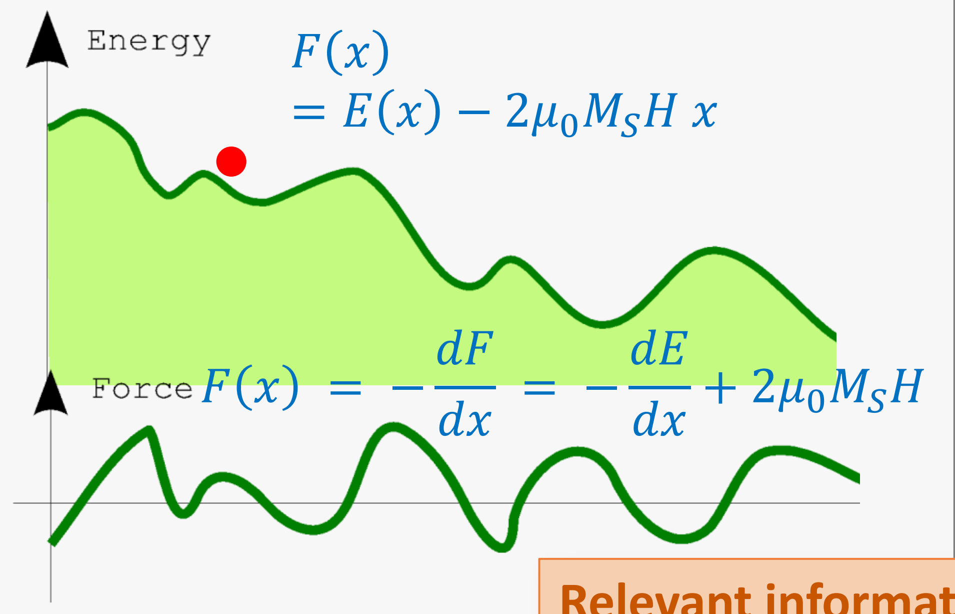
Pinning of domain walls

Example : domain wall to be moved along a 1d system

Without applied field



With applied field



E. Kondorski, On the nature of coercive force and irreversible changes in magnetisation, Phys. Z. Sowjetunion 11, 597 (1937)

Relevant information

- Microstructure
- Chemical composition
- Crystal structure

Precessional dynamics and switching

LLG equation

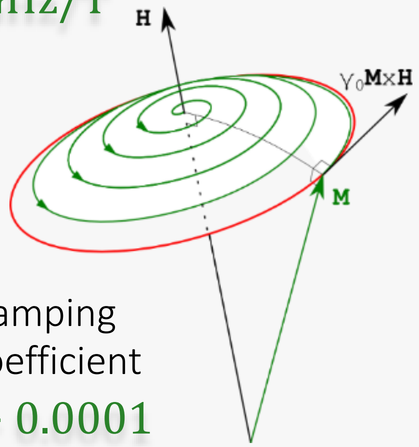
- Describes: precessional dynamics of magnetic moments
- Applies to magnetization, with phenomenological damping

$$\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t} = -|\gamma_0|\mathbf{m}\times\mathbf{H} + \alpha\,\mathbf{m}\times\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t}$$

$\gamma_0 = \mu_0\gamma < 0$ Gyromagnetic ratio

$\gamma_s = 28\ \mathrm{GHz/T}$

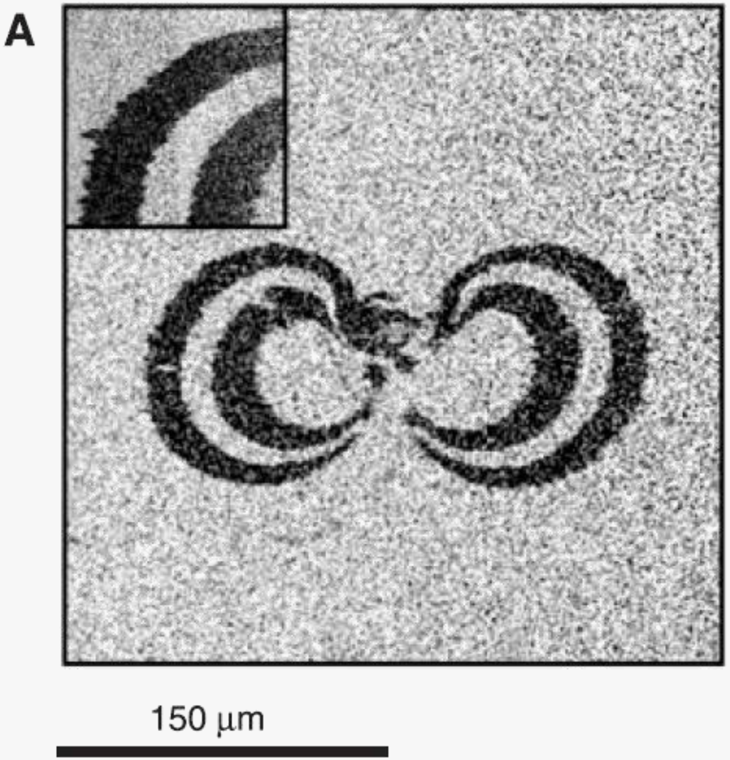
Larmor precession



$\alpha > 0$ Damping coefficient

$\alpha = 0.1 - 0.0001$

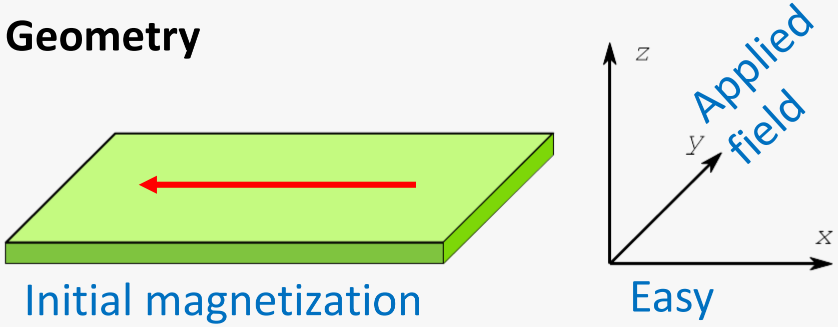
Pioneering experiment of precessional magnetization reversal



C. Back et al., Science 285, 864 (1999)

Precessional trajectories

Geometry

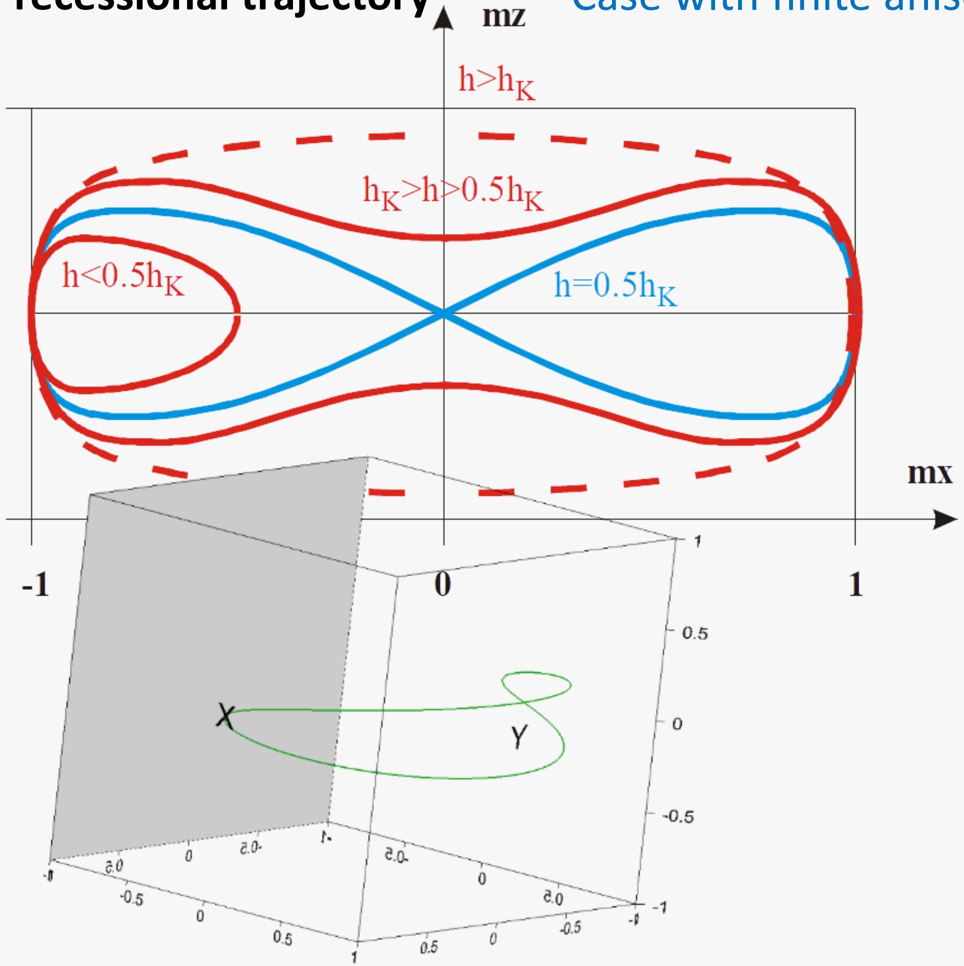


$$\frac{d\mathbf{m}}{dt} = -|\gamma_0|\mathbf{m} \times \mathbf{H} + \text{damping}$$

- Precession around its own demagnetizing field
- Threshold for switching is half the Stoner-Wohlfarth one

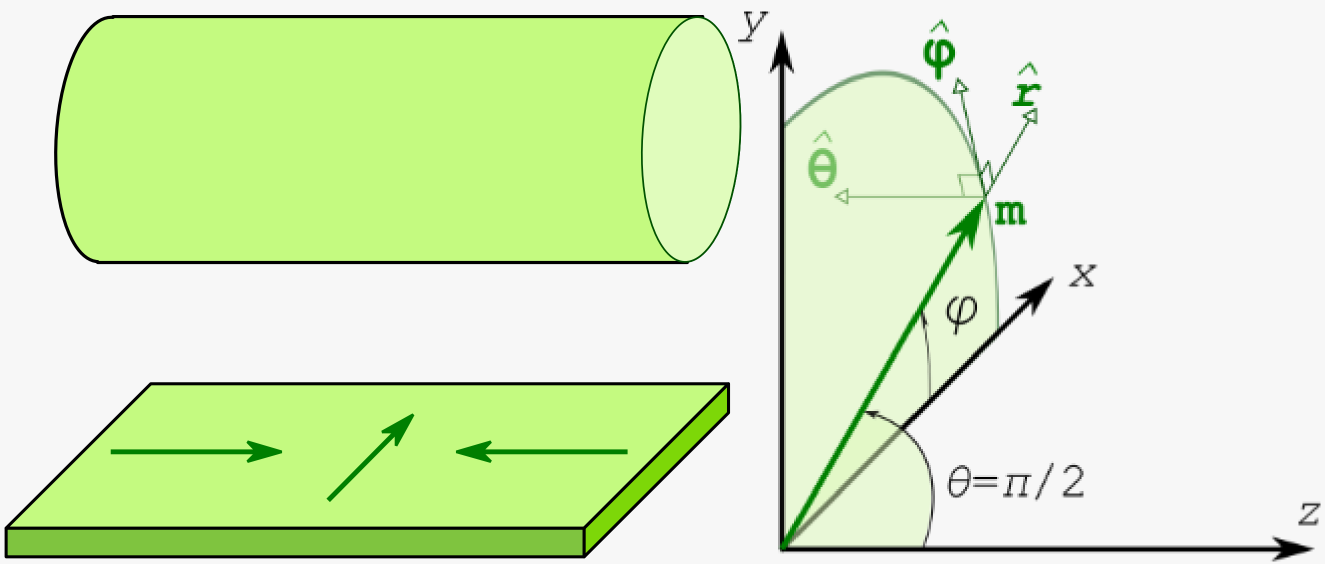
Precessional trajectory

Case with finite anisotropy



Precessional motion of magnetic domain walls

Precessional dynamics of transverse walls under magnetic field



Precessional dynamics under magnetic field

$$\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t} = -|\gamma_0|\mathbf{m}\times\mathbf{H} + \alpha\,\mathbf{m}\times\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t}$$

A. Thiaville, Y. Nakatani, Domain-wall dynamics in nanowires and nanostrips, in *Spin dynamics in confined magnetic structures {III}*, Springer (2006)

Wall speed

$$v = \alpha|\gamma_0|\Delta H$$

$$v = |\gamma_0|\Delta H/\alpha$$

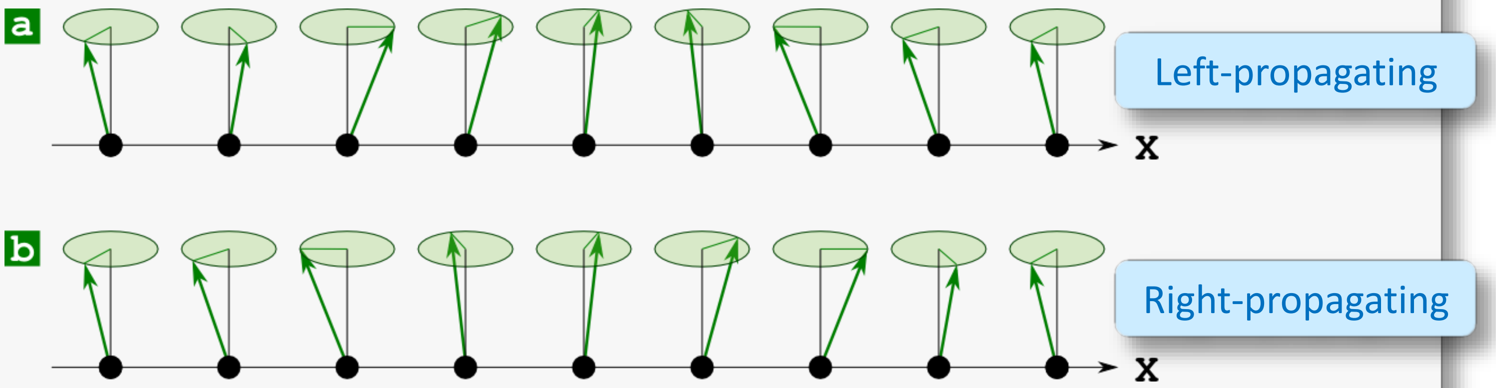
- Walker field $H_W = \alpha M_s/2$ $\approx \text{few} \quad \text{mT}$
- Walker speed $v = |\gamma_0|M_s\Delta/2$

$\text{up to} \approx 100\ \text{m/s}, \text{to km/s}$

Spin waves

Propagating Larmor precession

- **Physics**: exchange promotes propagation
- Spin waves have an angular frequency ω and a vector for propagation
- **There exist various geometries**, related to the direction of $\mathbf{M}$ versus $\mathbf{k}$, and the geometry of the system (thin film etc.)



Dispersion curve

- **Physics**: exchange implies additional energy, and thus higher frequency

$$\omega(k) = \omega_0 + Dk^2$$

D Spin-wave stiffness coefficient

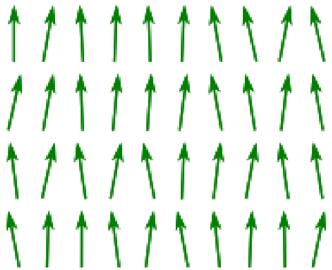
- **Dipolar energy**: depending on the spin-wave geometry, dipolar energy provides additional contributions to D, possibly with a negative value.

Situations for spin waves

- **Thermally-excited** → Contributes to the decay of magnetization with temperature
- **Magnonics:** excited on purpose using a radio-frequency field or a spin-polarized current.

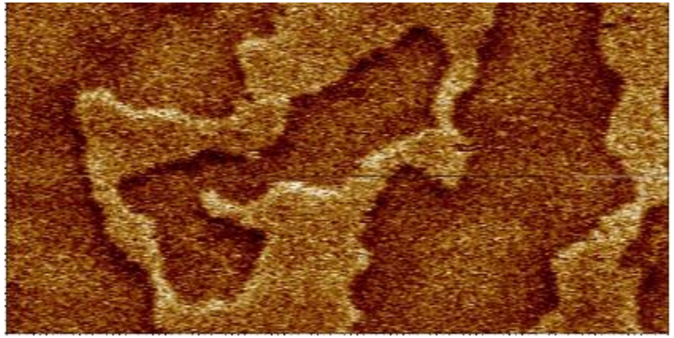
Table of contents

- Magnetic ordering

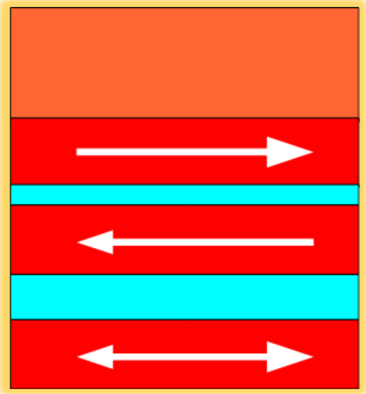


- Magnetic anisotropy

- Domains and magnetization processes

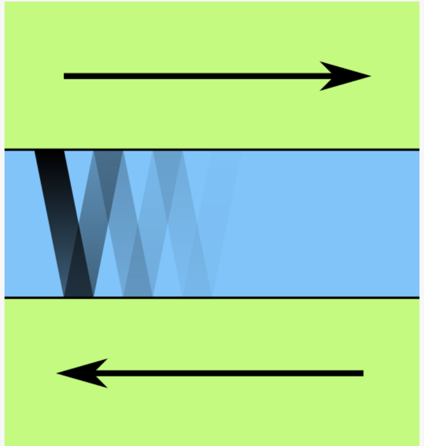


- Magnetic stacks



Interlayer exchange coupling

Spin-dependent quantum confinement



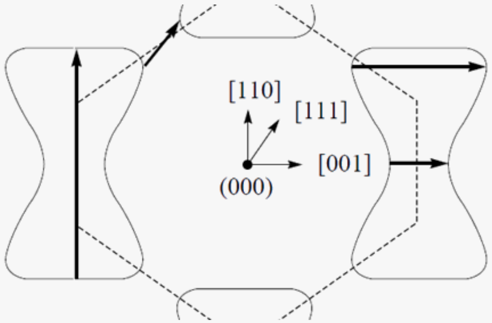
- Forth & back phase shift

$$\Delta\varphi = qt + \varphi_A + \varphi_B$$

- Spin dependence:

$$r_A, \varphi_A, r_B, \varphi_B$$

Oscillating constructive and destructive interferences with spacer thickness



Cu Fermi surface

- Importance of nesting
- Depends on crystal direction

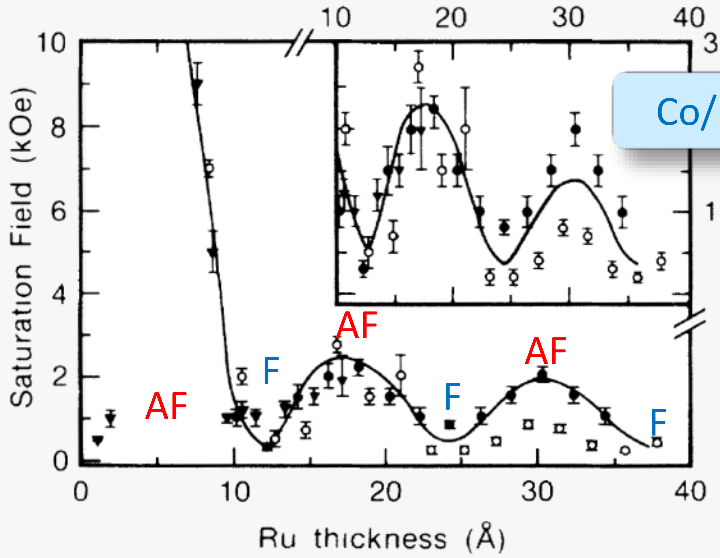
P. Bruno, J. Phys. Condens. Matter 11, 9403 (1999)

Coupling strength

$E_s(t) = J(t)\cos\theta$ with unit: $\mathrm{J/m^2}$

$$\theta = \langle \mathbf{m}_1, \mathbf{m}_2 \rangle$$

$$J(t) = \frac{A}{t^2}\sin(q_\alpha t + \Psi)$$

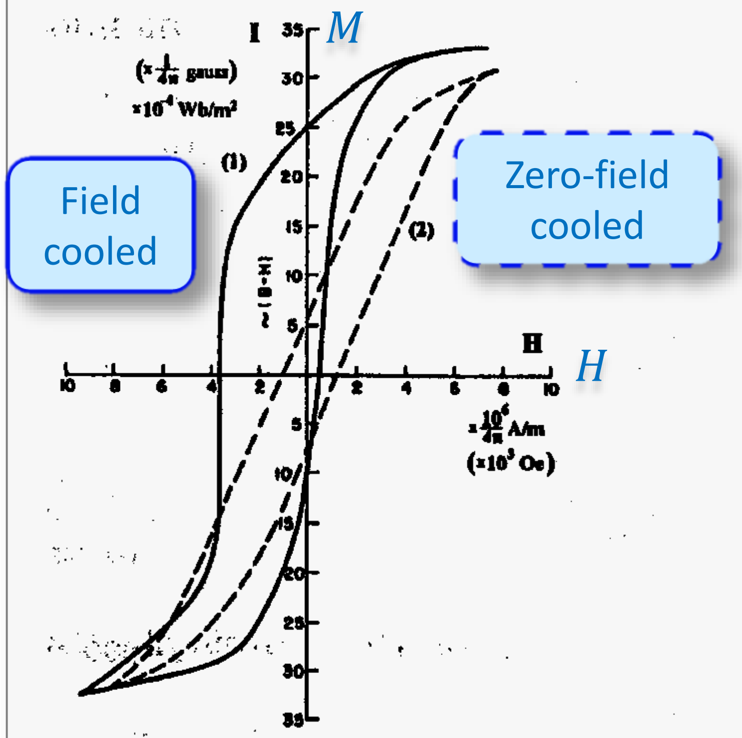


S. S. P. Parkin et al., PRL64, 2304 (1990)

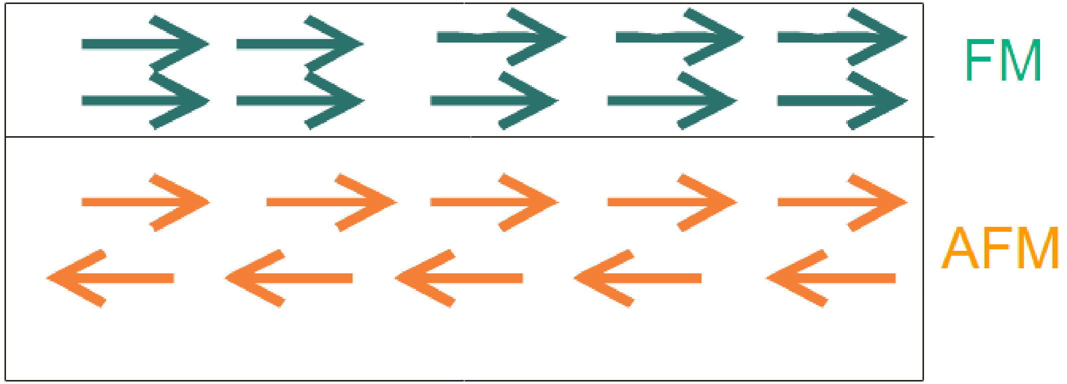
- RKKY = Ruderman-Kittel-Kasuya-Yoshida
- A function quantum effect at room temperature !
- Crucial to couple magnetic layers in stacks

Exchange bias

Seminal invertigation



Meiklejohn and Bean, Phys. Rev. 102, 1413 (1956), Phys. Rev. 105, 904, (1957)



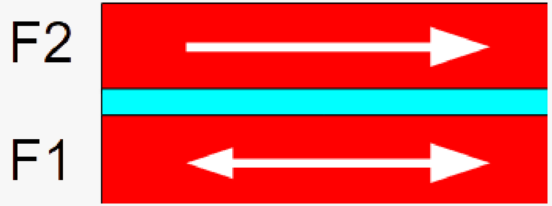
- Field-shift of hysteresis loop
- Increase of coercivity
- Crucial to design reference layer in memories

Exchange bias, J. Nogués and Ivan K. Schuller, J. Magn. Magn. Mater. 192 (1999) 203

Exchange anisotropy—a review, A E Berkowitz and K Takano, J. Magn. Magn. Mater. 200 (1999)

Synthetic antiferromagnets and spin valves

RKKY Synthetic Ferrimagnets (SyF) – Basics



- Crude phenomenology

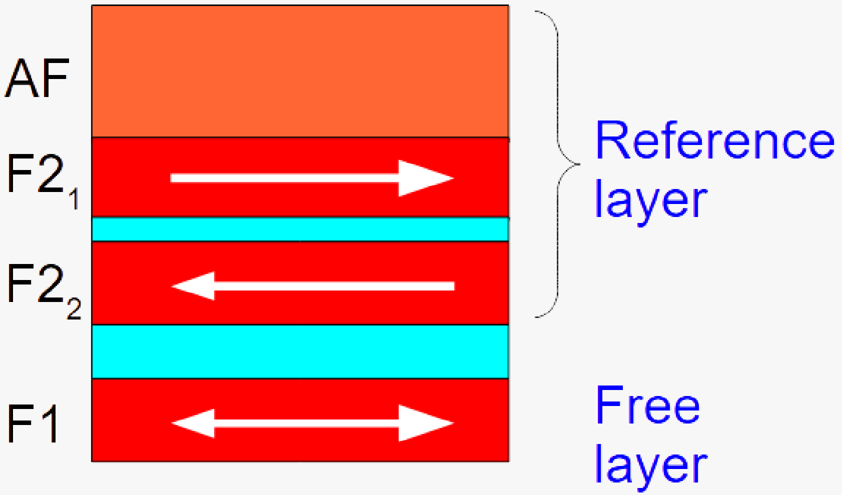
$$M = \frac{|e_1 M_1 - e_2 M_2|}{e_1 + e_2} \qquad K = \frac{e_1 K_1 + e_2 K_2}{e_1 + e_2}$$

$$\Rightarrow H_c \approx \frac{e_1 M_1 H_{c,1} + e_2 M_2 H_{c,2}}{|e_1 M_1 - e_2 M_2|}$$

- Enhances coercivity
- Reduces cross-talk in dense arrays

Spin valves

- "Free" and reference layers



B. Diény et al., J. Magn. Magn. Mater. 93, 101 (1991)

- Spin-valves are key elements in magneto-resistive devices (sensors, memories)
- Control Ru thickness within the Angström !


"That's all Folks!"