Fields, moments, units

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Our interest in magnetism, and organizing ESM

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Sent to esm@magnetism.eu on 12 Sep.2010

Dear Institute,

I've always had a fascination with electromagnetism, and have pondered the theories of gravity. One thing I've come across in preliminary research is that the current theories largely fail to include human element in, as if we're just baseless objects trapped here without a role in the ultimate reason. (...)

Humans are magnets, too, as we possess iron. (…) If you take two magnets, they stick together when proper polars are placed near each other. What causes humans to act as the 2nd magnet in gravity is the iron found in humans. Earth, obviously the big magnet with the most iron, is able to control humans, the far smaller magnet with less iron. (…) Ultimately there is one controlling magnet for the entire universe somewhere in space holding it all together, like Galileo said.

Calculations of Earth's maximum gravitation pull could be made by testing individual boosters on humans and converting the thrust needed into some kind of formula which returns Earth's magnetic energical pull. (…) While it doesn't conclude why other things on Earth are in the same situation as us, it is also based on magnetism and humans have to have their own role in the matter.

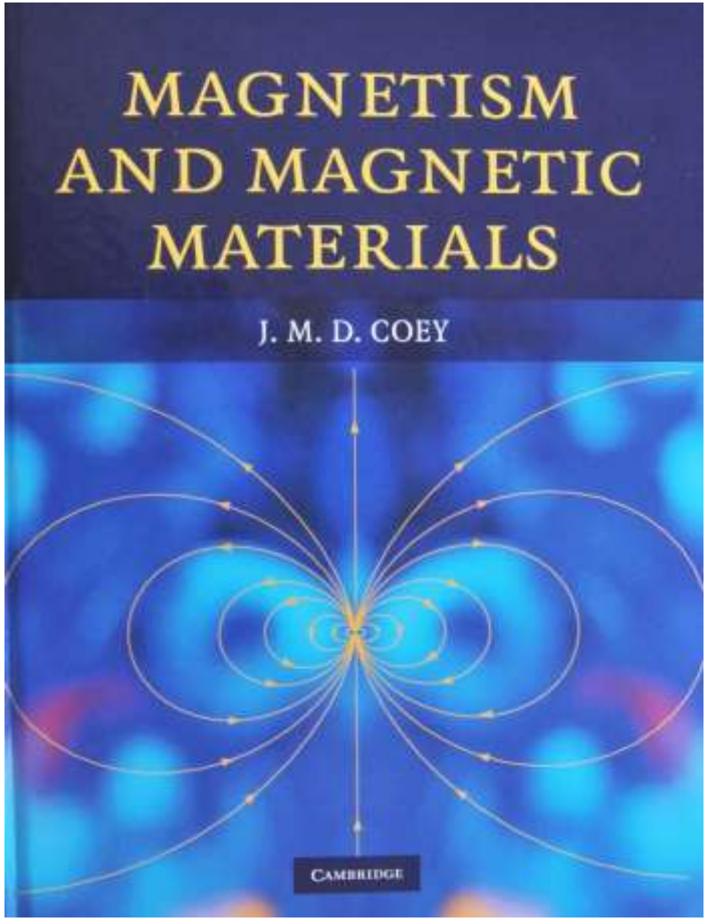
Further research into it needs to be done as these are very preliminary original thoughts.

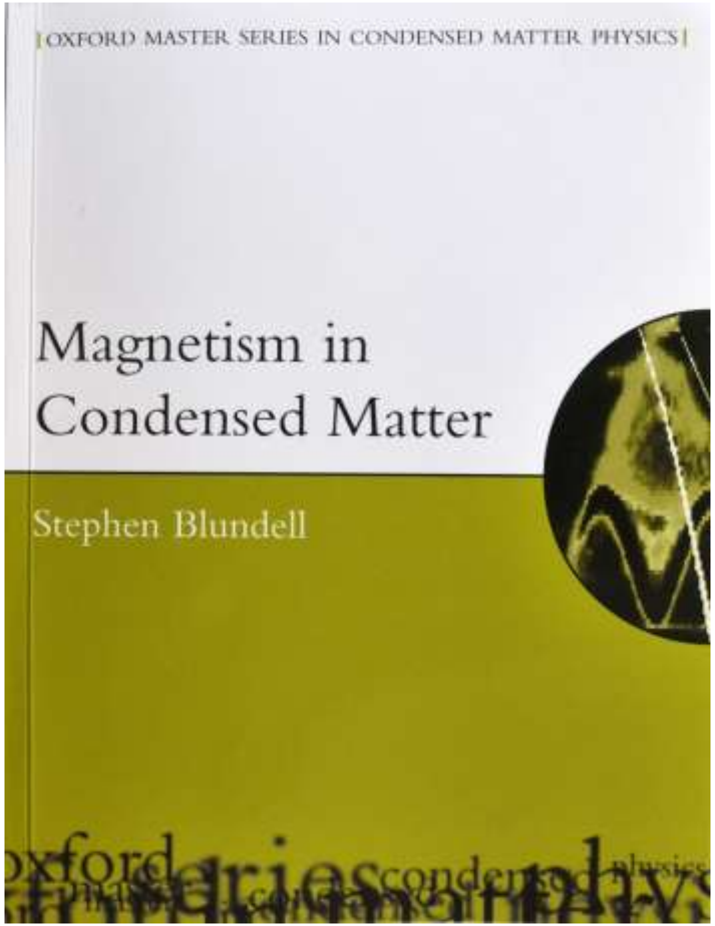
Regards,

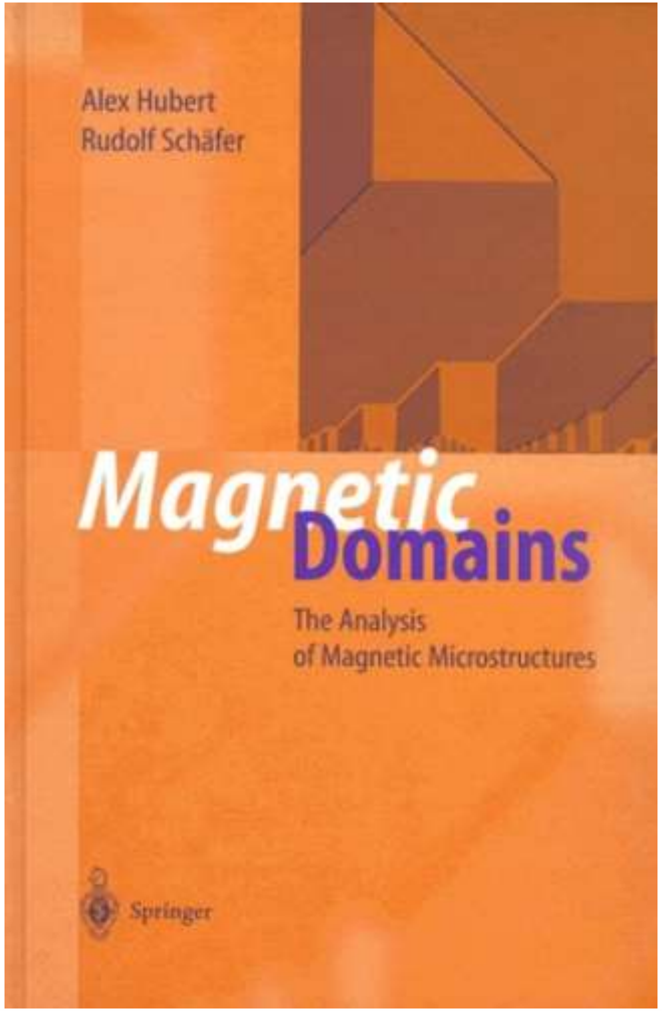
XXX YYY.

Understand the deep roots of magnetism

Some references







What is a quantity?

What is a unit ?

Quantities and units in physics

Quantity

- Example: speed $\mathbf{v} = \delta\boldsymbol{\ell}/\delta t$
- Dimension: $\dim(\mathbf{v}) = \mathrm{L}\cdot\mathrm{T}^{-1}$

Units

- Why?
  - Provide a measure
  - Universality: share with others
- Possible formalism:

$$X = X_\alpha \langle X\rangle_\alpha$$

Quantity

Measure

Reference quantity

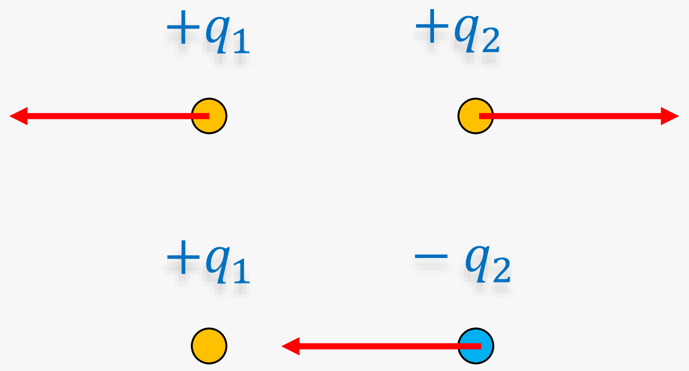
$$\langle L\rangle_{\mathrm{SI}} = \mathrm{meter} = 100\langle L\rangle_{\mathrm{cgs}}$$

$$L = 50\langle L\rangle_{\mathrm{SI}} = 5000\langle L\rangle_{\mathrm{cgs}}$$

The electric charge and the electric field

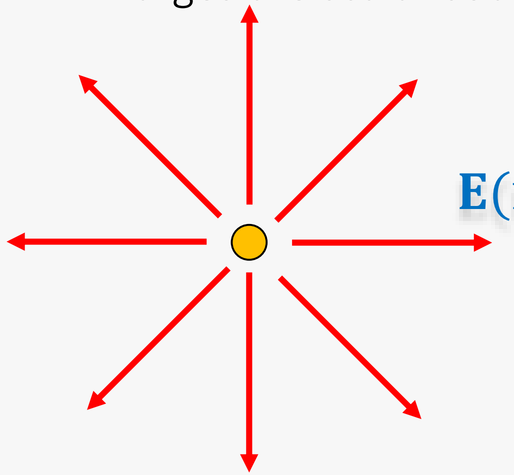
Facts: interaction between charges

$$\mathbf{F}_{1\to 2} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}^2}\mathbf{u}_{12}$$



Modeling by the Physicist

- Electric field $\mathbf{E}_{1\to 2}$ $\quad\mathbf{F}_{1\to 2} = \mathrm{q}_2\mathbf{E}_{1\to 2}$
- Charges are scalar sources of electric field



$$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0 r^2}\mathbf{u}$$

The electric charge and the electric field

Microscopic level: Maxwell equation

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\rho = \frac{\delta Q}{\delta \mathcal{V}}$$ Volume density of electric charge

Macroscopic level: Gauss theroem

- Ostogradski theorem

$$\iiint_{\mathcal{V}} \boldsymbol{\nabla} \cdot \mathbf{E} \, \mathrm{d}\mathcal{V} = \oiint_{\partial \mathcal{V}} \mathbf{E} \cdot \mathbf{n} \, \mathrm{d}\mathcal{S}$$

$$\Rightarrow \frac{Q}{\epsilon_0} = \iiint_{\mathcal{V}} \frac{\rho}{\epsilon_0} \, \mathrm{d}\mathcal{V} = \oiint_{\partial \mathcal{V}} \mathbf{E} \cdot \mathbf{n} \, \mathrm{d}\mathcal{S}$$

Link

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \ldots = \frac{E_x(x + \delta x) - E_x(x)}{\delta x} + \cdots$$

Origin of magnetic interactions

Century-old facts

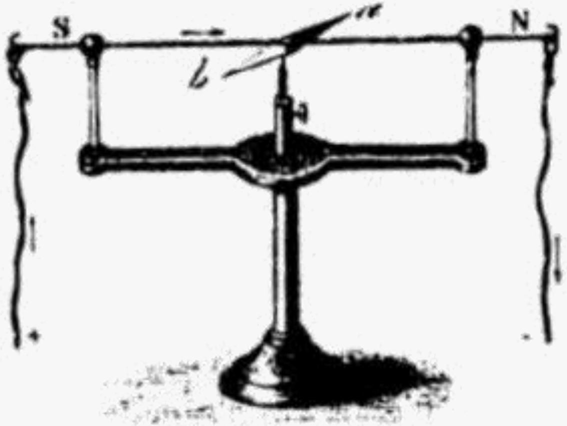
- Magnetic materials (rocks)


Magnetite


Light-struck

- Magnetic field of the earth

Œrsted experiment in 1820




Hans-Christian Oersted, 1777–1851.

→ Birth of electromagnetism

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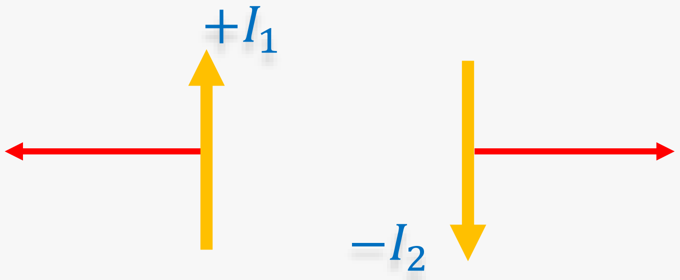
Facts: interaction between charge currents

$$\delta \mathbf{F}_{1\rightarrow 2} = \mu_0 \frac{I_1 I_2 [\delta \boldsymbol{\ell}_2 \times (\delta \boldsymbol{\ell}_1 \times \mathbf{u}_{12})]}{4\pi r_{12}^2}$$

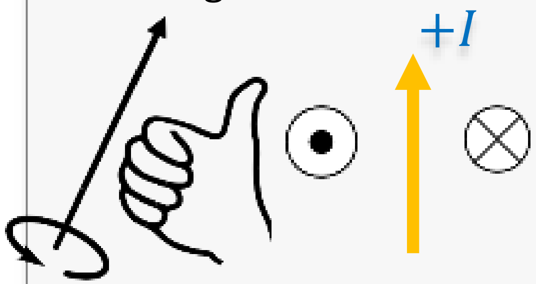


Note: former definition of the Ampère:

The force between two infinitely wires 1m apart with current 1A is $2 \times 10^{-7}\ \mathrm{N/m}$

Modeling by the Physicist

- Magnetic induction field: Biot & Savart law



$$\delta \mathbf{B} = \frac{\mu_0 I \delta \boldsymbol{\ell} \times \mathbf{u}}{4\pi r^2}$$

- Retrieve the force (Laplace)

$$\delta \mathbf{F}_2 = I_2 \delta \boldsymbol{\ell} \times \mathbf{B}(\mathbf{r}_2)$$

$$\Rightarrow \mathbf{F} = q\, \mathbf{v} \times \mathbf{B}$$

- Magnetic induction field defined through Lorentz Force

The electric current and the magnetic induction field

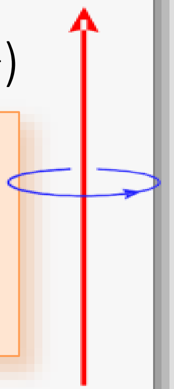
Microscopic level: Maxwell equation

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$$

j: Volume density of current (A/m²)

- J is the vectorial source of curl of B

Unit for B: tesla (T)



Macroscopic level: Ampere theorem

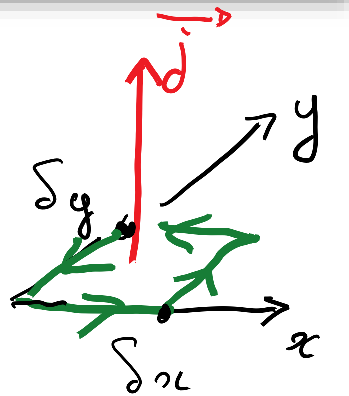
- Stokes theorem

$$\iint_{\mathcal{S}} (\nabla \times \mathbf{B}) \cdot \mathbf{n} \, \mathrm{d}\mathcal{S} = \oint_{\partial \mathcal{S}} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell}$$

$$\Rightarrow \quad I = \mu_0 \iint_{\mathcal{S}} (\mathbf{j} \cdot \mathbf{n}) \, \mathrm{d}\mathcal{S} = \oint_{\partial \mathcal{S}} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell}$$

Link

$$\nabla \times \mathbf{B} = \begin{pmatrix} \cdots \\ \cdots \\ \dfrac{\partial B_y}{\partial x} - \dfrac{\partial B_x}{\partial y} \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \dfrac{B_y(x+\delta x) - B_y(x)}{\delta x} - \dfrac{B_x(y+\delta y) - B_x(y)}{\partial y} \end{pmatrix}$$



Maxwell equations (in vacuum)

$\nabla \cdot \mathbf{E} = \dfrac{\rho}{\epsilon_0}$	Gauss theorem
$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	Faraday law of induction
$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} \right)$	Ampère theorem
$\nabla \cdot \mathbf{B} = 0$	B is divergence free (no magnetic poles)

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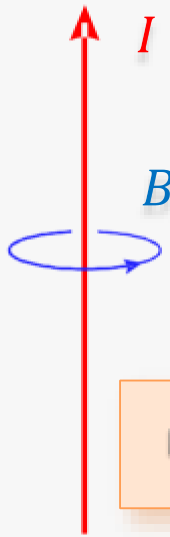
Biot and Savart

$$\delta\mathbf{B} = \frac{\mu_0 I \delta\boldsymbol{\ell} \times \mathbf{u}}{4\pi r^2}$$

- Note: $1/r^2$ decay

Ampere theorem and Œrsted field

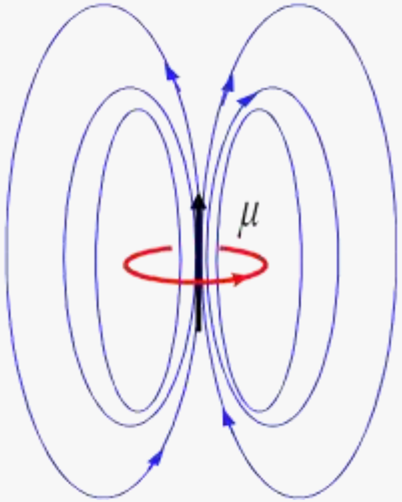


$$B_\theta = \frac{\mu_0 I}{2\pi r}$$

Integrate

- Note: $1/r$ decay

The magnetic point dipole



- Simple loop

$\boldsymbol{\mu} = I\mathcal{S}\,\mathbf{n}$ **Unit**: $\mathrm{A}\cdot\mathrm{m}^2$

- General definition

$$\boldsymbol{\mu} = \frac{1}{2}\iiint_{\mathcal{V}} \mathbf{r} \times \mathbf{j}(\mathbf{r})\,\mathrm{d}\mathcal{V}$$

Derive

$$\mathbf{B} = \frac{\mu_0}{4\pi r^3}\left[\frac{3}{r^2}(\boldsymbol{\mu}\cdot\mathbf{r})\mathbf{r} - \boldsymbol{\mu}\right]$$

- Note: $1/r^3$ decay

$$\mathbf{B} = \frac{\mu_0}{4\pi r^3}\left(2\mu\cos\theta\,\mathbf{u}_r + \mu\sin\theta\,\mathbf{u}_\theta\right)$$

The magnetic point dipole in a magnetic induction field

Energy

$\mathcal{E} = -\boldsymbol{\mu} \cdot \mathbf{B}$ Zeeman energy (J)

Demonstration

- Work to compensate Lenz law during rise of B
- Integrate torque from Laplace force while flipping dipole in B

Force

$\mathbf{F} = \boldsymbol{\mu} \cdot (\overline{\overline{\boldsymbol{\nabla}\mathbf{B}}})$

- Valid only for fixed dipole
- No force in uniform magnetic induction field

Torque

$$\boldsymbol{\Gamma} = \oint \mathbf{r} \times I(\mathrm{d}\boldsymbol{\ell} \times \mathbf{B}) = \boldsymbol{\mu} \times \mathbf{B}$$

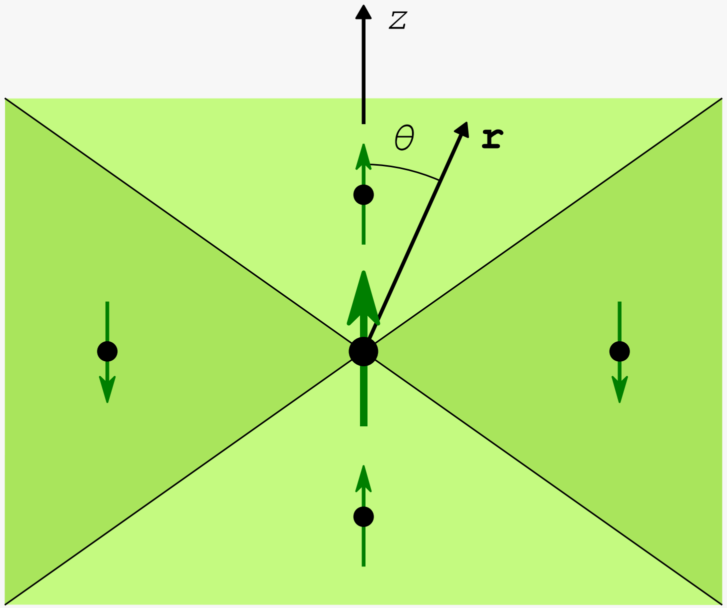
- Inducing precession of dipole around the field
- It is energy-conservative, as expected from Laplace (Lorentz) force

Two interacting magnetic point dipoles

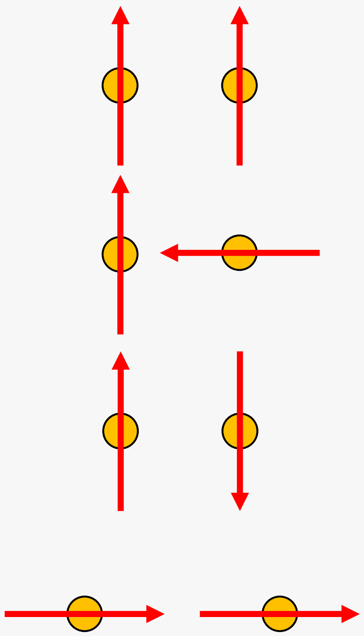
Energy

$$\mathcal{E} = -\frac{\mu_0}{4\pi r^3}\left[\frac{3}{r^2}(\boldsymbol{\mu}_1\cdot\mathbf{r})(\boldsymbol{\mu}_2\cdot\mathbf{r}) - \boldsymbol{\mu}_1\cdot\boldsymbol{\mu}_2\right]$$

- The dipole-dipole interaction is anisotropic



Examples



$$\mathcal{E} = +2\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

$$\mathcal{E} = +\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

$$\mathcal{E} = 0$$

$$\mathcal{E} = -\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

$$\mathcal{E} = -2\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

Magnetization

Definition

- Volume density of magnetic point dipoles

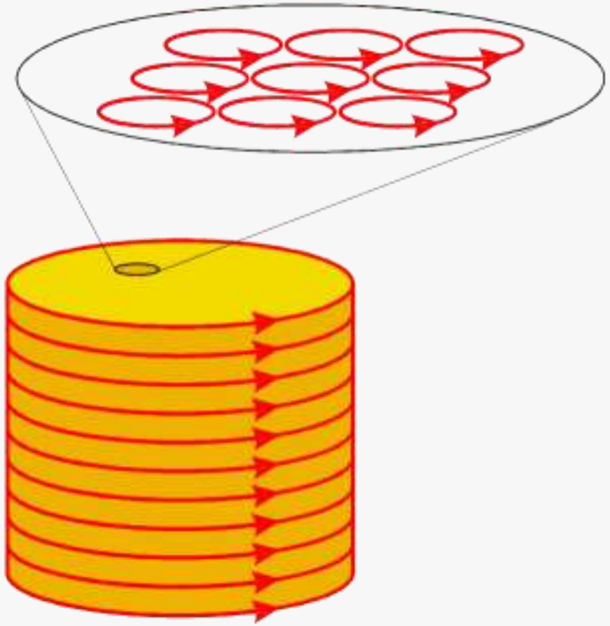
$$\mathbf{M} = \frac{\delta\boldsymbol{\mu}}{\delta\mathcal{V}} \qquad \mathrm{A/m}$$

- Total magnetic moment of a body

$$\boldsymbol{\mathcal{M}} = \int_{\mathcal{V}} \mathbf{M}\,\mathrm{d}\mathcal{V} \qquad \mathrm{A\cdot m^2}$$

- Applies to: ferromagnets, paramagnets, diamagnets etc.
- Must be defined at a length scale much larger than atoms
- Is the basis for the micromagnetic theory

Equivalence with surface currents



- Name: Amperian description of magnetism
- Surface current equals magnetization $\mathrm{A/m}$

Free currents and bound currents

Back to Maxwell equations

- Disregard fast time dependence: magnetostatics

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \cancel{\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}} \right)$$

- Consider separately real charge current, $\mathbf{j}_\mathrm{c}$ from fictitious currents of magnetic dipoles $\mathbf{j}_\mathrm{m}$

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 (\mathbf{j}_\mathrm{c} + \mathbf{j}_\mathrm{m})$$

- One can show: $\boldsymbol{\nabla} \times \mathbf{M} = \mathbf{j}_\mathrm{m}$ $\quad \mathrm{A/m^2}$

$\mathbf{M} \times \mathbf{n} = \mathbf{j}_\mathrm{m,s}$ $\quad \mathrm{A/m}$

- Outside matter, $\mathbf{B}$ and $\mu_0 \mathbf{H}$ coincide and have exactly the same meaning.

The magnetic field H

- One has: $\boldsymbol{\nabla} \times \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) = \mathbf{j}_\mathrm{c}$

- By definition: $\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$ $\quad \mathrm{A/m}$

$$\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{j}_\mathrm{c}$$

B versus H : definition of the system

- M: local (infinitesimal) part in $\delta\mathcal{V}$ of the system defined when considering a magnetic material

- H: The remaining of B coming from outside $\delta\mathcal{V}$, liable to interact with the system

Derivation of the dipolar field

The dipolar field $\mathbf{H}_\mathrm{d}$

- By definition: the contribution to **H** not related to free currents (possible to split as Maxwell equations are linear)

$$\boldsymbol{\nabla}\times\mathbf{H}_\mathrm{d}=0 \Rightarrow \mathbf{H}_\mathrm{d}=-\nabla\phi_\mathrm{d}$$

$$\mathbf{H}=\mathbf{H}_\mathrm{d}+\mathbf{H}_\mathrm{app}$$

External to magnetic body

Analogy with electrostatics

$$\boldsymbol{\nabla}\times\mathbf{E}=0 \Rightarrow \mathbf{E}=-\nabla\phi$$

Derive the dipolar field

Maxwell equation $\boldsymbol{\nabla}\cdot\mathbf{B}=\mathbf{0} \rightarrow \boldsymbol{\nabla}\cdot\mathbf{H}_\mathrm{d}=-\boldsymbol{\nabla}\cdot\mathbf{M}$

$$\Rightarrow \mathbf{H}_\mathrm{d}(\mathbf{r})=-M_\mathrm{s}\iiint_{\mathcal{V}'}\frac{[\boldsymbol{\nabla}\cdot\mathbf{m}(\mathbf{r}')]\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{V}'$$

To lift the singularity that may arise at boundaries, a volume integration around the boundaries yields:

$$\mathbf{H}_\mathrm{d}(\mathbf{r})=\iiint\frac{\rho(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{V}'+\oiint\frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$$

$\rho(\mathbf{r})=-M_\mathrm{s}\,\boldsymbol{\nabla}\cdot\mathbf{m}(\mathbf{r})$ → volume density of magnetic charges

$\sigma(\mathbf{r})=M_\mathrm{s}\,\mathbf{m}(\mathbf{r})\cdot\mathbf{n}(\mathbf{r})$ → surface density of magnetic charges

Stray field and demagnetizing field

Vocabulary

- Generic names

  Magnetostatic field

  Dipolar field

- Inside material

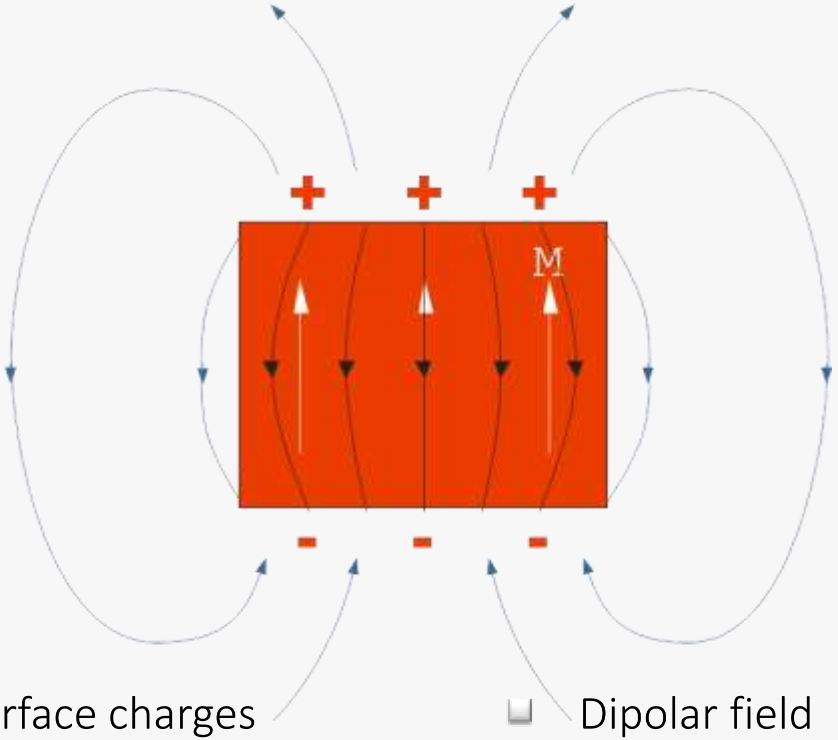
  Demagnetizing field

- Oustide material

  Stray field

Example

Permanent magnet (uniformly-magnetized)



- Surface charges

$$\sigma(\mathbf{r})=M_\mathrm{s}\mathbf{m}(\mathbf{r})\cdot\mathbf{n}(\mathbf{r})$$

- Dipolar field

$$\mathbf{H}_\mathrm{d}(\mathbf{r})=\oiint\frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$$

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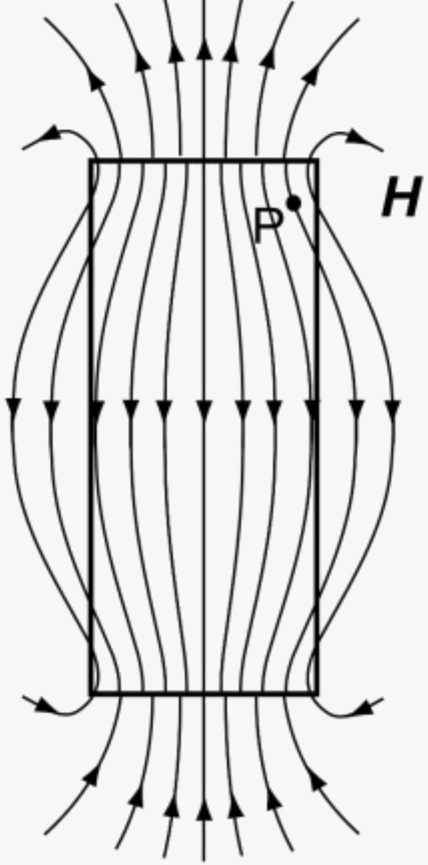
Coulombian

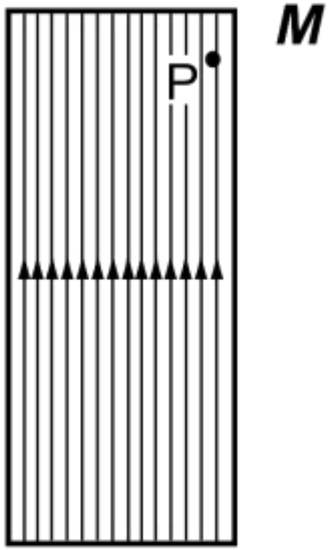
- Pseudo-charges source of Hd



$\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{0}$

- No closed lines
- $\Delta H_{\parallel} = 0$
- $\Delta \mathbf{H} \cdot \mathbf{n} = \sigma$

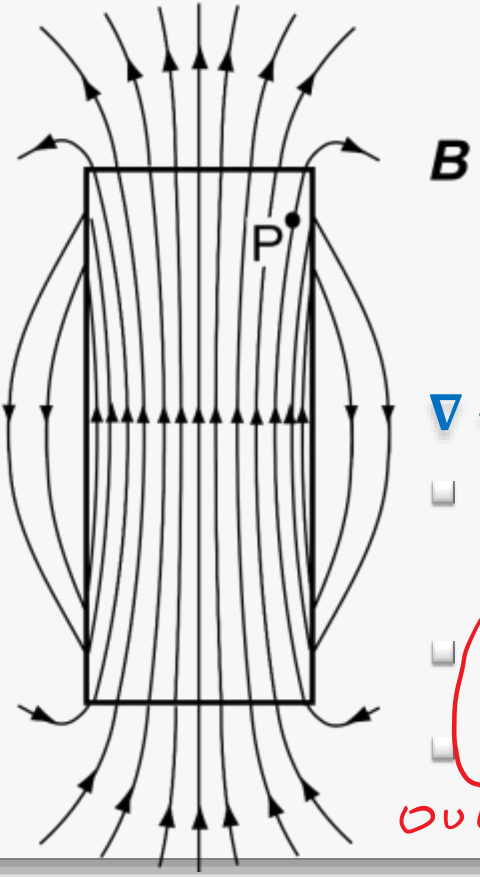
out - in



From: M. Coey's book

Amperian

- Fictitious currents source of B



$\boldsymbol{\nabla} \cdot \mathbf{B} = \mathbf{0}$

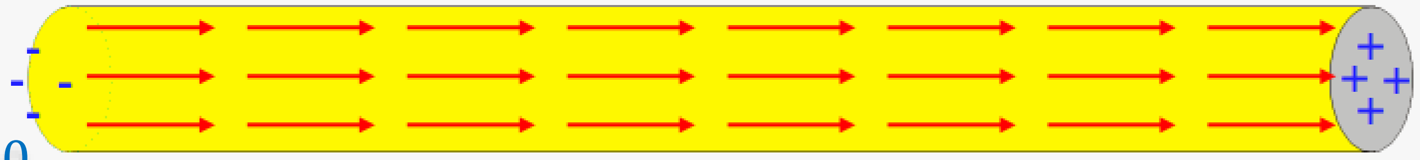
- No magnetic monopole
- $\Delta B_{\perp} = 0$
- $\Delta \mathbf{B} = \mu_0 \mathbf{j} \times \mathbf{n}$

out - in

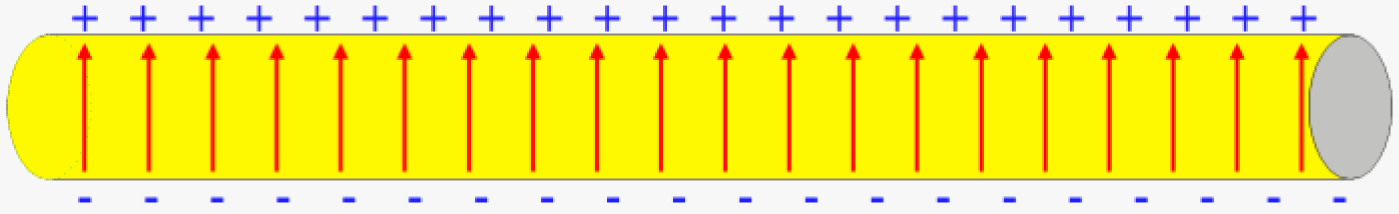
Examples of magnetic charges

The long cylinder

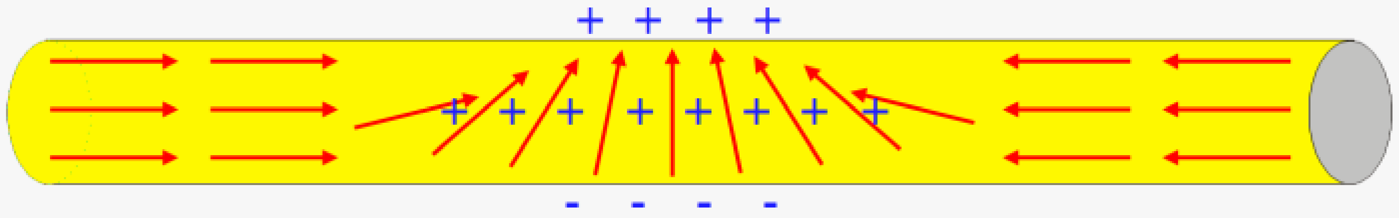
- Note for infinite cylinder: no charge $\mathcal{E}_{\mathrm{d}} = 0$



- Charges on side surfaces



- Surface and volume charges



Take-away message

- Dipolar energy favors alignment of magnetization with longest direction of sample

Dipolar energy

- Zeeman energy of microscopic volume

$$\delta\mathcal{E}_Z = -\mu_0 \mathbf{M}\delta\mathcal{V} \cdot \mathbf{H}_{ext}$$

- Elementary volume of a macroscopic system creating its own dipolar field

$$E_d = \delta\mathcal{E}_d/\delta\mathcal{V} = -\frac{1}{2}\mu_0 \mathbf{M} \cdot \mathbf{H}_d$$

mutual energy

- Total energy of macroscopic body

$$\mathcal{E}_d = -\frac{1}{2}\mu_0 \iiint_{\mathcal{V}} \mathbf{M} \cdot \mathbf{H}_d \, d\mathcal{V}$$

$$\mathcal{E}_d = \frac{1}{2}\mu_0 \iiint_{\mathcal{V}} \mathbf{H}_d^2 \, d\mathcal{V}$$

- Always positive. Zero means minimum

Size considerations

$$\mathbf{H}_d(\mathbf{r}) = \text{Volume} + \oiint \frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3} d\mathcal{S}'$$

- Unchanged if all lengths are scaled: homothetic. Check that the following is a solid angle:

$$d\boldsymbol{\Omega} = \frac{(\mathbf{r}-\mathbf{r}')d\mathcal{S}'}{|\mathbf{r}-\mathbf{r}'|^3}$$

- H_d does not depend on the size of the body
- Neither does the volume density of energy
- Said to be a long-range interaction

Range of dipolar interactions in low dimensionality

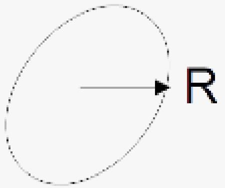
Range

- Upper bound of dipolar field

$$\|\mathbf{H}_\mathrm{d}(\mathbf{r})\| \leq M_\mathrm{s} t \int \frac{2\pi r}{r^3} \mathrm{dr}$$

Integration

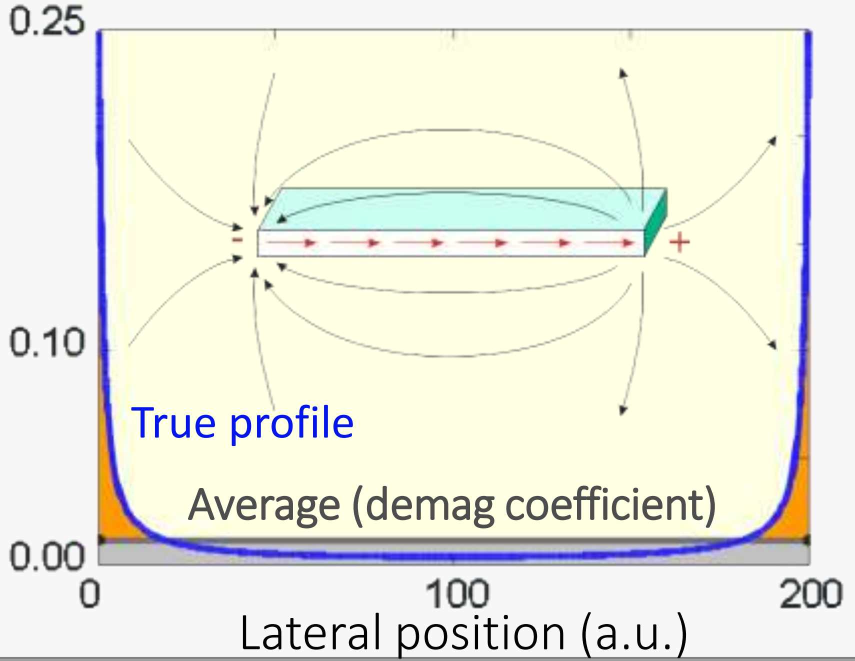
Hd for dipole



$$\|\mathbf{H}_\mathrm{d}(\mathbf{R})\| \leq \mathrm{Cste} + \mathcal{O}(1/R)$$

Non-homogeneity

- Example: flat strip with aspect ratio 0.0125



- Dipolar fields are short-ranged in low dimensionality
- Dipolar fields are highly non-homogeneous in large aspect ratio systems
- Consequences: non-uniform magnetization switching, excitation modes etc.

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Dipolar energy for uniform magnetization

$$\mathbf{M}(\mathbf{r}) = \mathbf{M} = M_\mathrm{s}(m_x\hat{\mathbf{x}} + m_y\hat{\mathbf{y}} + m_z\hat{\mathbf{z}})$$

Implicit $\sum_{i=x,y,z}$

- No volume charges: $\rho(\mathbf{r}) = -M_\mathrm{s}\,\boldsymbol{\nabla}\cdot\mathbf{m}(\mathbf{r}) = 0$
- Dipolar field: $$\mathbf{H}_\mathrm{d}(\mathbf{r}) = \oiint_{\partial\mathcal{V}} \frac{[\mathbf{M}(\mathbf{r}')\cdot\mathbf{n}(\mathbf{r}')]\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}' = M_\mathrm{s} m_i \oiint_{\partial\mathcal{V}} \frac{\mathrm{n}_i(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$$
- Dipolar energy:

$$\mathcal{E}_\mathrm{d} = -\frac{1}{2}\mu_0 \iiint_{\mathcal{V}} \mathbf{M}(\mathbf{r})\cdot\mathbf{H}_\mathrm{d}(\mathbf{r})\mathrm{d}\mathcal{V} = -\frac{1}{2}\mu_0 M_s^2 m_i \iiint_{\mathcal{V}} \mathrm{d}\mathcal{V} \oiint_{\partial\mathcal{V}} \frac{n_i(\mathbf{r}')\,\mathbf{m}\cdot(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$$

$$\mathcal{E}_\mathrm{d} = -K_\mathrm{d} m_i m_j \iiint_{\mathcal{V}} \mathrm{d}\mathcal{V} \oiint_{\partial\mathcal{V}} \frac{n_i(\mathbf{r}')\left(r_j - \mathrm{r}_\mathrm{j}'\right)}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$$

Implicit $\sum_i \sum_{j=x,y,z}$

$$\langle\mathbf{H}_\mathbf{d}(\mathbf{r})\rangle = -M_\mathrm{s}\,\bar{\bar{\mathbf{N}}}\cdot\mathbf{m}$$

$$\mathcal{E}_\mathrm{d} = K_\mathrm{d} V\,\mathbf{m}\cdot\bar{\bar{\mathbf{N}}}\cdot\mathbf{m}$$

See more detailed approach: M. Beleggia et al., JMMM 263, L1-9 (2003)

Demagnetizing coefficients – Take-away messages

For any shape of body

$$\langle \mathbf{H_d}(\mathbf{r}) \rangle = -M_s\, \bar{\bar{\mathbf{N}}} \cdot \mathbf{m}$$

$$\mathcal{E}_\mathrm{d} = K_\mathrm{d} V\, \mathbf{m} \cdot \bar{\bar{\mathbf{N}}} \cdot \mathbf{m}$$

Dipolar anisotropy is always of second order

- $\bar{\bar{\mathbf{N}}}$ demagnetizing tensor. Always positive, and can be diagonalized. $N_x + N_y + N_z = 1$

$$\mathcal{E}_\mathrm{d} = K_\mathrm{d} V \left(N_x m_x^2 + N_y m_y^2 + N_z m_z^2 \right)$$

- Along main directions

$$\langle H_{\mathrm{d},i}(\mathbf{r}) \rangle = -N_i M_\mathrm{s}$$

Hypothesis uniform M may be too strong
Remember: dipolar field is NOT uniform

For ellipsoids etc.

- Condition: boundary is a polynomial of the coordinates, with degree at most two

Slabs (thin films), cylinders, ellipsoids

$$z^2 = \left(\frac{t}{2}\right)^2 \qquad \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \qquad \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$

$$\mathbf{H_d} = -M_s\, \bar{\bar{\mathbf{N}}} \cdot \mathbf{m}$$

$$\mathcal{E}_\mathrm{d} = K_\mathrm{d} V\, \mathbf{m} \cdot \bar{\bar{\mathbf{N}}} \cdot \mathbf{m}$$

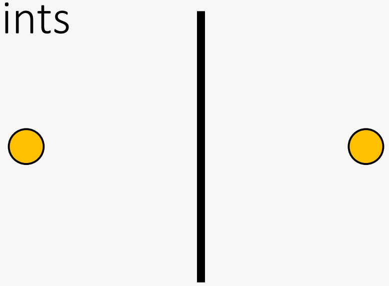
- Along main directions

$$H_{\mathrm{d},i} = -N_i M_\mathrm{s}$$

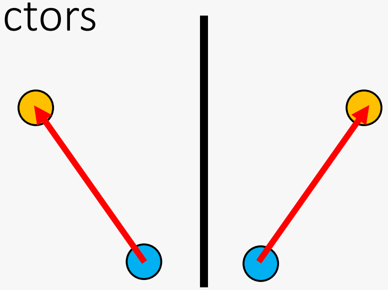
M and H may not be colinear along non-main directions

Reminder about plane symmetry

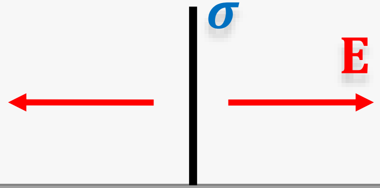
- Points



- Vectors



- Example: electric field $\nabla \cdot \mathbf{E} = \dfrac{\rho}{\epsilon_0}$



Magnetic fields are pseudo-vectors

- Curl is a chiral operator

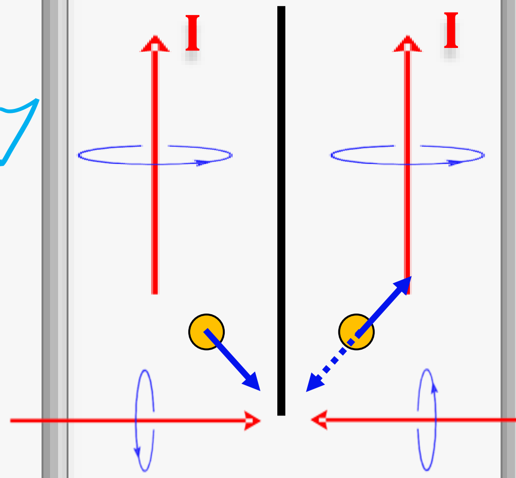
Antisymmetry

$$\delta \mathbf{B} = \frac{\mu_0 I \delta \boldsymbol{\ell} \times \mathbf{u}_{12}}{4\pi r^2}$$

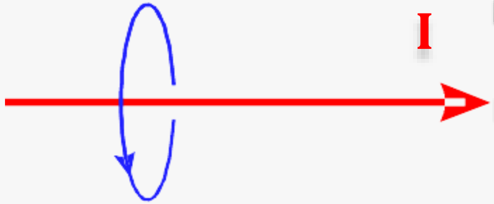
Symmetry

Antisymmetry

B is antisymmetric



What use? Example: Ampere theorem and Œrsted field



- Symmetry of I with plane containing I
- Antisymmetry of B: is azimuthal $\boldsymbol{\ell}$

$$\Rightarrow \quad B_\theta = \frac{\mu_0 I}{2\pi r}$$

Time inversion symmetry

Time inversion symmetry of Maxwell equations

- What happens with operation $t \to -t$

Unchanged $\nabla \cdot \mathbf{E} = \dfrac{\rho}{\epsilon_0}$

Unchanged $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ → $-\mathbf{B}$, → $-t$

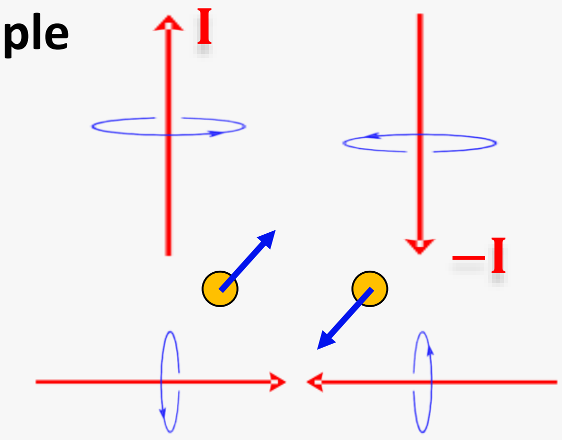
Inversed $\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} \right)$

$-\mathbf{B}$ ← ; → $-\mathbf{j}$; → $-t$

Inversed $\nabla \cdot \mathbf{B} = 0$ → $-\mathbf{B}$

- Maxwell equations remains valid
- Solutions must comply with time-reversal symmetry

Example



What use? Magneto-crystalline anisotropy

$$E(\theta) = \cancel{K_{10} \cos\theta} + \cancel{K_{01} \sin\theta} + K_{11} \cos\theta \sin\theta + K_{20} \cos^2\theta + K_{02} \sin^2\theta + \cancel{K_{30} \cos^3\theta} + \cancel{K_{03} \sin^3\theta} + \cancel{K_{21} \cos^2\theta \sin\theta} + \cancel{K_{12} \cos\theta \sin^2\theta} + \cdots$$

- Odd terms are forbidden

Units

Definitions

	SI system		cgs-Gauss	
	Meter	m	Centimeter	cm
	Kilogram	kg	Gram	g
	Second	s	Second	s
	Ampere	A	Ab-Ampere	ab-A = 10A

$\mathbf{B}=\mu_0(\mathbf{H}+\mathbf{M})$ $\quad$ $\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}$

$\mu_0=4\pi\times10^{-7}$ SI $\quad$ $\mu_0=4\pi$

Conversions

Field	$\mathbf{H}$	$1\ \mathrm{A/m}$	↔	$4\pi\times10^{-3}\ \mathrm{Oe}$ (Oersted)
Moment	$\boldsymbol{\mu}$	$1\ \mathrm{A.m^2}$	↔	$10^3\ \mathrm{emu}$
Magnetization	$\mathbf{M}$	$1\ \mathrm{A/m}$	↔	$10^{-3}\ \mathrm{emu/cm^3}$
Induction	$\mathbf{B}$	$1\ \mathrm{T}$	↔	$10^4\ \mathrm{G}$ (Gauss)
Susceptibility	$\chi=M/H$	1	↔	$1/4\pi$

Problems with cgs-Gauss

- The quantity for charge current is missing

  No check for homogeneity

  Mix of units in spintronics

- Inconsistent definition of H

  Dimensionless quantities are effected: demag factors, susceptibility etc.

- More in the practical on units

Define quantities

- Times
- Length
- Mass
- Electric charge

Fixed values

- Speed of light -> Define meter
- Planck constant -> Defines kg
- Charge of the electron

To be measured

- Magnetic permeability of vacuum

$\mu_0 \neq 4\pi \times 10^{-7}$ S. I.

$$\mu_0 = 4\pi[1 + 2.0(2.3) \cdot 10^{-10}] \times 10^{-7} \text{ S. I.}$$

Thank you for your attention !

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